Edgy yet HOW aerodynamic, really?

by M. Crescimanno

Here is one example article, followed by a series of questions that we raised and then how to answer them using what you learned in this course, and some other ”found” or ”inspired” numbers.

In this report I use the numbers that were given in an article about aerodynamic efficiency of automobile shapes and relate it to actual changes in gas mileage and other associated questions. I do so using basic physical principles and some known and found numbers. Because I am using this formally for a class, and there are copyright protections, I have not reproduced and re-typeset the article in the back of this report in its entirety...you would normally include a copy of the article on paper that you would submit with your report.

QUESTIONS:

In this report we address the following questions raised by our interests and our reading of the article,

1) What is the typical drag coefficient for a car and what does a reduction of .02 mean percentage-wise.

2) Does the statement ”at highway speeds, 60% of the energy is used to overcome air resistance” lead to an estimate of the typical drag coefficient. What other information would you need to know to relate this fact to an actual typical drag coefficient?

3) They say that with the Delta C = -.02 improvement, the mileage increased to 24 miles per gallon. What was the mileage before the improvement?

4) Why is the air pressure higher lower to the ground as a car moves by?

5) How much greater is the air pressure there, say at 60 Mi/Hr.

6) Do any of these data indicate what is the most efficient speed to drive at? Can you compute that most efficient speed?

ANALYSIS NARRATIVE: (ANSWERS TO THE QUESTIONS:)

1) What is the typical drag coefficient for a car and what does a reduction of .02 mean percentage-wise.

For a typical ”production car” one has $C_v = 0.30$ (From ”http://www.hypercars.com/main.html”) 

For specialty cars, the $C_v$’s can be somewhat lower,

$C_v = 0.19$ EV-1
$C_v = 0.16$ Precept
\[ C_v = 0.20 \quad \text{Opel Eco Speedster} \]
\[ C_v = 0.20 \quad \text{Honda IMAS} \]

A drag coefficient reduction of .02 is probably a decrease in the 10 percent range. Not too shabby, but in the light of the coefficients of some of the specialty cars, this doesn’t look so terribly hard to achieve.

2) Does the statement ”at highway speeds, 60% of the energy is used to overcome air resistance” lead to an estimate of the typical drag coefficient. What other information would you need to know to relate this fact to an actual typical drag coefficient?

The drag has chiefly two pieces—a rolling friction term proportional to the load on the tires (i.e. the weight of the vehicle) and an air resistance term.

\[ F_d = F_{tire} + F_{air} = C_r M g + C_v \rho v^2 A \quad (1) \]

Note that both \( C_r \) and \( C_v \) are dimensionless (\( \rho \) is the density of air, assume about 1 Kg/m\(^3\)). Note that the drag term due to speed increases rapidly at higher speeds. \( C_r \) is called the restitution loss coefficient. It is this coefficient that you can reduce somewhat when you overpressurize your tires a bit (not recommended in snow and/or sandy roads).

Thus, knowing this form for the drag force, the mass of the car, the front projected area \( A \), and using the statement from the article that ”at highway speeds, 60% of the energy is used to overcome air resistance” does indeed lead to an estimate of a typical \( C_r \). An numerical example, take the ford Pinto; ”at highway speeds, 60% of the energy is used to overcome air resistance” does indeed lead to an estimate of a typical \( C_r \). An numerical example, take the ford Pinto; (see (from http://en.wikipedia.org/Ford_Pinto)) It weighed about 2000 lbs (910Kg), had a 56 kW engine (delivering 40 kW to the wheels) and could go from 0 to 60 mph in 10.8 seconds. I couldn’t find a measured value for the projected front area of the car, but I am estimating that it was about 2 meters wide by about 1.7 meters high, leading to 3.4 m\(^2\) projected front area. With these numbers, I find a value for \( C_r \) using that 65 mph (highway speed) = 28.9 m/s,

\[ C_r M g/(C_v \rho v^2 A) = .4/.6 \quad (2) \]

So that \( C_r = .064 \), where we assumed that the pinto was not designed for low \( C_v \), and so took \( C_v = .3 \). This is lower than the friction coefficient of things sliding on ice, but in the same ballpark.

Let’s check and see if this makes sense. Suppose that you are moving at 5 mph, like in a shopping mall parking lot. If you go into neutral and coast to rest how far would this predict that you would go?

At this small a speed, since the drag force goes as the square of the velocity, we can effectively ignore it here and just assume that the net restitution loss brings us to rest. Thus, \( F_d = C_r M g = 568 \) N, and thus our acceleration would be \( F_d/M = .624m/s^2 \). Now, if we were moving at 5 mph (=2.22 m/s) we would then come to rest in \( t = v/a = 2.22/.624 \) 4 seconds, during which time we would have gone \( d = vt + 1/2at^2 = (\text{remember that this } a \text{ is negative}) \) 5 meters, or about 15 feet. That is pretty reasonable.

3) They say that with the \( \Delta C = -.02 \) improvement, the mileage increased to 24 miles per gallon. What was the mileage before the improvement?

Now we relate the drag reduction and the reported improved miles per gallon (mpg) to arrive at an estimate of the original mpg.
Note that they are referring to the mpg’s at highway speeds. Note also that the $F_{\text{tires}}$ hasn’t changed, just the $C_v$ has. To progress further we will have to make an assumption about the original $C_v$. But let’s first develop a logical narrative to simplify the problem.

Note that the miles you’ve driven are $x = vt$ and the gallons you consumed of gas are proportional to the engine’s power multiplied by the time it ran, that is, $\text{gallons} = kPt$ where ‘$k$’ is a constant of proportionality. Thus, the mpg = $\frac{x}{\text{gallons}} = \frac{vt}{kPt} = 1/kF_d$.

We want to find $mpg_0$, the original mpg given that the present $mpg = 24$, all at the same ‘highway’ velocity for comparison. So, flipping the relation for the mpg above, we have the two equations,

$$\frac{k'}{mpg_0} = F_{\text{tires}} + C_v(old)A\rho v^2$$

$$\frac{k'}{24} = F_{\text{tires}} + C_v(new)A\rho v^2$$

where we are assume that the only difference is the change in the $C_v$’s (and $k = 1/k'$ is how efficiently the car turns gas into force at the wheels and also $F_{\text{tires}}$, the restitution loss of the tires are exactly the same). Now, since we are told that 60% of the drag force at highway speeds is due to the air drag term (before the improvement...clearly that went down with the reduction of the $C_v$) that means

$$F_{\text{tires}} = .4/6(C_v(old)A\rho v^2)$$

$$\frac{k'}{mpg_0} = 1./6k(C_v(old)A\rho v^2)$$

and so we plug these into the ”$k'/24 =...$” equation above to get

$$\frac{k'}{24} = \frac{k'}{mpg_0}(.4 + .6 * C_v(new)/C_v(old))$$

and thus, were we to assume that $C_v(old) = .3$ and the $C_v(new) = .3 - .02 = .28$, which is the reduction that the quote, then from the above equation we learn that

$$mpg_0 = 23$$

So, with this change they have increased the mpg of the car by 1 mpg. That doesn’t sound like terribly much, but it is! it is impressive that the shape of the car can matter as much as a mile per gallon.

4) Why is the air pressure higher lower to the ground as a car moves by?

This is a qualitative question. The air pressure is apparently higher underneath a moving car. Presumably this is due to the downdraft of air that was formerly infront of the car, but is then displaced by the car. The downdraft apparently pressurizes slightly the underside of the car, much like blowing into a plastic bag increases the pressure enough to inflate the bag. This is what I think they call ‘ram pressure’ or ‘stagnation pressure’ as in a dynamic pressure differential caused by the wake in the flow. That is, the air is being swept out of the way, but just like the water hump in the front of a moving boat, there is a little buildup of air in the front of the moving car.

5) How much greater is the air pressure there, say at 60 Mi/Hr.

We now estimate the pressure increase underneath the car. A simple way is with the Bernoulli principle (Chapter 14...I needed to look ahead, and you may too!) which basically is energy conservation for each little element of air along the air flow lines. This indicates that

$$P - P_0 = \frac{1}{2} \rho v^2 = 415n/m^2$$
assuming that the air velocity stagnates to essentially zero underneath the car. Since one atmosphere is 10130 n/m², this doesn’t look so enormous, but over the 7.5 m² (estimated) area of the car underside, this means that the car is effectively as much as 415 n/m² × 7.5 m² / 9.8 m/s² = 317 Kg lighter due to the cushion of air that it is riding on.

That looks like a good chunk of the weight of the car (910 kg). Two comments; this is an overestimate since the underside of the car is open, the air is moving and so the pressures don’t ever get quite that high. But, note that it increases rapidly with the velocity, so this can be an important effect at larger velocities, which is one reason that race cars, particularly Indy 500 cars are designed to minimize the pressure that builds up below the car; They are moving fast enough to literally ‘fly’ and that would not be conducive to making tight turns!

6) Do any of these data indicate what is the most efficient speed to drive at? Can you compute that most efficient speed?

This last question is indeed a bit more open ended. From our simple model since we have mpg as being proportional to the reciprocal of the $F_d$, we will find our largest mpg at the minimum $F_d$, that is, zero velocity! Clearly something is missing here...

One way to make this a sensible problem is to go back and examine the assumptions that went into the mpg derivation. Note that the in our derivation we assumed that $v/kP$ could be simplified via $P = vF_d$. But, as the wiki article points out, the engine’s rated mechanical power is 56 Kw but only 40 Kw of that makes it to the wheels (to then be dissipated as $vF_d$). This means that $vF_d$ is the power dissipated at the wheels, and to that we must add an offset $P_{trans}$ of energy lost through the transmission/drivetrain.

This $P_{trans}$ doesn’t vanish at zero speed...recall that you have to burn some gas while idling. Thus, the total $mpg = \frac{v}{P_{trans} + vF_d}$ vanishes at $v = 0$. It also vanishes at large $v$. Thus, since it is always positive, there is some value for $v$ in between 0 and infinity in which the $mpg$ has a maximum.

If we knew what $P_{trans}$ was a function of $v$ we could differentiate the mpg formula above wrt $v$ and set it to zero to find this maximum. In actual practice, cars are designed to have their maximum mpg at about 39 mpg.

Note that including this more detailed formula for the mpg will reduce the effect of the shape of the car on the mpg. That is, the real change in mpg associated with the reduction of $C_v$ by .02 is probably significantly less than what we found, 1 mpg. That may be why the article doesn’t give that actual number. An increase in mileage of, say, .7 mpg doesn’t sound that ‘sexy’ when folks are buying foreign cars that are doing 20 mpg better than Detroit’s wrecks.
Bibliography

1) Edgy, yet still Aerodynamic, by PHIL PATTON NYTimes, Automobile section, Dec. 20, 2008
2) http://www.hypercars.com/main.html
3) http://en.wikipedia.org/Ford_Pinto
Edgy, yet still Aerodynamic

NYTimes, Automobile section, Dec. 20, 2008

By PHIL PATTON, Published: December 19, 2008

WAYNE KOESTER was pleasantly surprised. Mr. Koester, who was a Ford aerodynamicist at the time, had been assigned to turn the popular boxy Fairlane design study that was introduced at the 2005 Detroit auto show, inspired by the woody station wagons of the 1940s, into the production Flex crossover. He had to produce a shape slippery enough to provide acceptable fuel mileage, and he feared the boxy show car would have to be radically revised.

Aerocorners include edges and bulges on the front of Toyota’s small city car, the iQ. To his surprise, in hundreds of tests at Ford’s Wind Tunnel 8 southwest of Detroit the original edges produced less drag than curved substitutes, Mr. Koester said.....