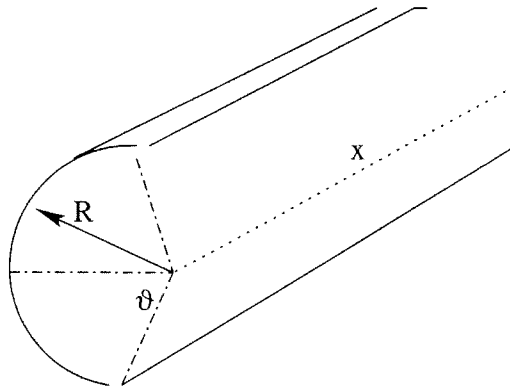


≡ KEY ≡

Yet more Coulomb Law Practice...potentials!

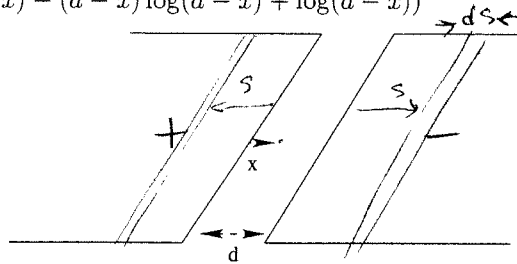
1) Show that the magnitude of the electric potential of a pipe-section as measured at the centerline of the pipe is given by $V = 4k\sigma R\theta \log(R)$ where θ is the angle that the pipe subtends and σ is its surface charge density.



$$\begin{aligned}
 V &= \int dV \\
 &= \int 2k\sigma R \ln R \\
 &= \int 2k\sigma R \ln R d\theta \\
 &= 2k\sigma R \ln R \int d\theta = 4k\sigma R \ln R
 \end{aligned}$$

$dA = R d\theta$

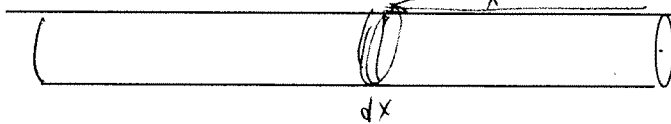
2) Show that the electric ~~field~~ ^{POTENTIAL} at a distance x between two oppositely-charged plates separated by distance d , each half-infinite and carrying surface charge of magnitude density σ is $V = 2k\sigma(x \log(x) - \log(x) - (d-x) \log(d-x) + \log(d-x))$



POT FROM LINE CHARGE $\sim 2k\lambda \ln R$

$$\begin{aligned}
 \lambda &= \sigma ds \\
 V &= \int dV = \int 2k\sigma ds \ln(s+d-x) - 2k\sigma ds \ln(s+x) \\
 &= \int 2k\sigma \ln\left(\frac{s+d-x}{s+x}\right) ds \\
 &= 2k\sigma \left[(s+d-x) \ln(s+d-x) - (s+x) \ln(s+x) \right] \\
 &= 2k\sigma \left[(d-x) \ln(d-x) - x \ln x \right]
 \end{aligned}$$

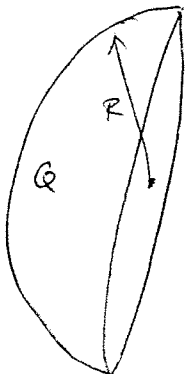
3) For a very thin wire of length L carrying total charge Q show that the electrostatic potential at a distance Z from the end as show is $V = (kQ/L) \log\left(\frac{L+Z}{Z}\right)$



$$\begin{aligned}
 V &= \int dV = \int \frac{k(\lambda dx)}{x+Z} = k\lambda \ln(x+Z) \Big|_0^L \\
 &= \frac{kQ}{L} \ln\left(\frac{L+Z}{Z}\right)
 \end{aligned}$$

$dq = \lambda dx$ $\lambda = Q/L$

4) For the half-sphere shown below, a charge Q is uniformly distributed across its surface. It has radius R . Show using Coulomb's law that the magnitude of the electric potential at X (the center of the sphere) is $\frac{kQ}{R}$ directed to the right.



$$\begin{aligned}
 V &= \int dV \quad dV = \frac{k dq}{R} \quad \text{ALL R'S THE SAME TO CENTER} \\
 \Rightarrow V &= \int dV = \frac{k}{R} \int dq = \frac{kQ}{R} \quad Q \text{ TOTAL CHARGE.}
 \end{aligned}$$