Don’t miss that boxed announcement on the last page!

and – from the field:

A student-faculty collaboration on the isotope shift

TeachSpin has long had a standing offer to make this Newsletter available to highlight achievements obtained by students using TeachSpin instrumentation ‘in the field’. Here’s an account of a faculty-student collaboration at Youngstown State University, using their TeachSpin ‘Diode-Laser Spectroscopy’ or DLS equipment to make a spectroscopic measurement of the isotope shift of the rubidium D2 line.

First, a bit of introduction. Tabletop optical spectroscopy has been revolutionized in the past generation by the invention of affordable and tunable diode-laser sources. A host of formerly textbook-only concepts have become real-life phenomena now accessible to hands-on discovery and control by students. Not just absorption and fluorescence, but also saturated absorption, polarization effects, Zeeman effects, resonant Faraday rotation, and the like are now phenomena your students can examine themselves in the advanced lab.

Next: what’s a ‘D2 line’? It all started with Fraunhofer’s arbitrary labelling (A, B, C, D, …) of ‘dark lines’, or absorption features, visible in spectrally-dispersed sunlight. Later came Kirchhoff’s discovery that each such absorption line corresponded to an emission line in some particular atomic species; Fraunhofer’s D-line (seen in absorption) coincides in wavelength with the bright-yellow emission line seen in terrestrial atomic sodium.

Next came wavelength measurement, and higher resolution; grating spectrometers allowed the assignment of an absolute wavelength (=589 nm) to the sodium D-line, and even a modest prism spectrometer can resolve that D-line into two lines (called D1 at 589.6 nm, D2 at 589.0 nm).

Of course there’s a quantized-energy-level interpretation of those transitions; it involves spin-orbit splitting in an excited $\ell = 1$ or P-state, giving a higher-in-energy $P_{\frac{3}{2}}$ state and a lower-in-energy $P_{\frac{1}{2}}$ state. Of the two closely-spaced D-lines, the transition coupling the $P_{\frac{3}{2}}$ excited state to the $S_{\frac{1}{2}}$ ground state in sodium (and in other alkali atoms) is the one of higher energy, shorter wavelength, and D2-labelling.

Rather than sodium, our DLS users are more familiar with atomic rubidium (Rb), where the D2 line falls in the near infrared at 780 nm, and the D1 line at longer wavelength (and lower energy) at 795 nm. And our DLS users also enjoy, not the part-per-thousand resolution that suffices to ‘split’ the sodium D-lines, but part-per-million resolution in studying the Rb D2-line.

At this level, there’s more to see – the D2 ‘line’ observed in absorption shows up as four features (see Fig. 1, next page). The wavelengths of all four transitions lie near 780 nm, meaning the four optical frequencies are all near $384 \times 10^{12}$ Hz or 384,000 GHz; but the four features are spread out over about 7 GHz in optical frequency. The two outer (and weaker) features are attributed to the $^{85}$Rb minority isotope, and the two inner features to the majority $^{87}$Rb isotope, occurring in natural rubidium. For each isotope, the ‘doublet’ that’s seen is attributable to ground-state hyperfine structure (hfs), the coupling of the nuclear spin magnetic moment to the spin of the atom’s valence electron.
Most users of our DLS system go on to use saturated-absorption techniques, by which the $D_2$-line transitions of Doppler-broadened spectral widths (about 0.5 GHz) can be resolved into Doppler-free individual components of width about 0.01 GHz, as seen above. These newly-revealed splittings are due to hyperfine effects in the upper, $P_{3/2}$, state.

Measuring absolute optical frequencies is still expensive and difficult, but users of our Fabry-Perot interferometer option can still assign an absolute frequency-difference scale to the scans they make over the $D_2$ spectrum of natural Rb. Two mirrors, spaced by separation $L$, create a series of optical resonances of wavelengths $\lambda_n$, labelled by integer $q$ and specified by $q \cdot \lambda_n = 2L$, the round-trip distance inside the Fabry-Perot cavity. (Here we ignore index-of-refraction issues.) Such resonant modes are equally-spaced in optical frequency $f_n$ with

$$f_{q+1} - f_q = \Delta f = \frac{c}{2L}.$$ 

A mirror spacing of order $L \approx 150 \text{ mm}$ gives a series of optical-frequency features with spacing $\Delta f \approx 0.5 \text{ GHz}$. These are shown (inverted) in the lowest trace in Fig. 1, as a series of very narrow maxima-in-transmission of the Fabry-Perot interferometer.

Using this sort of ‘comb’ of features equally spaced in optical frequency, it is possible to assign correct frequency differences (though not the absolute optical frequencies) to rubidium transitions seen in any sort of spectroscopic scan. Many a student has used this technique to measure, for example, the hyperfine structure splitting, in both isotopes of Rb, and for both the ground ($S_{1/2}$) state and the excited ($P_{3/2}$) state. Alternatively, the ground-state hfs splitting, very well known via direct transitions involving microwave-frequency measurements, can be used to calibrate the tooth spacing of the Fabry-Perot comb.

Students Jonathan Feigert, Ted Bucci, Brandon Chamberlain, Alex Giovannone, and instructor Mike Crescimanno, of Youngstown State University, have made yet another application of this sort of frequency-measurement capability. Every feature of the Rb spectrum can be assigned a location on the ‘ruler’ of Fabry-Perot features, and assigned a location in the form of $N.n$ ‘teeth of the comb’. Here $N$ is an integer obtained by counting whole teeth, and fraction $n$ ($0 \leq n < 1$) is obtained by interpolation.

[Aside: This technique thus relies on the known-to-be-equal spacing of the teeth of the comb, and it replaces the somewhat non-linear scan of laser frequency vs. time, or vs. command-voltage ramp – a non-linearity attributable to the piezoelectric actuator which tilts the frequency-controlling diffraction grating in the diode-laser head.]

In addition to giving (separately for each isotope) the location of each of the hfs components of the atoms’ transitions, this technique can establish the isotope shift of the $D_2$ line. To extract that, we’d like to know where on the frequency scale the $^{85}\text{Rb}$ and the $^{87}\text{Rb}$ $D_2$ transitions would lie, if hyperfine-structure splittings were to be ‘dialed to zero’. So first, by taking a multiplicity-weighted average over individual components, one can create the pseudo-frequencies that would
arise in the limit of vanishing upper-state hfs. Another such multiplicity-weighted average can create, for each isotope, the pseudo-frequency that would arise in the limit of vanishing ground-state hfs. (Example: Due to $2F+1$ multiplicity, in $^{87}\text{Rb}$ the transitions from the $F=2$ ground state get weight 5/8, while those from the $F=1$ ground state get weight 3/8, in forming the right average.) And the frequencies thus established for the two isotopes’ $D_2$ transitions need not, and in fact do not, coincide! That’s the ‘isotope shift’ of the $D_2$ transitions.

Crescimanno’s students measure this shift (for the $D_2$ line of Rb) to be 84 MHz (or 0.084 GHz), which is about 17% of one tooth-spacing of the Fabry-Perot comb. So the effect is not marginal, nor even small, compared to the available resolution for optical frequency differences. The isotope shift is small, however, compared to the absolute optical frequency:

$$84\text{ MHz} = 0.22 \times 10^{-6} \text{, a fifth of a part per million.}$$

Isotope shifts of this sort are best-known in hydrogen-like atoms, in which the ‘reduced-mass effect’ dominates the shift. For example, the red Balmer-$\alpha$ line for an infinitely-massive nucleus would have frequency

$$hf = E_3 - E_2 = \hbar c \, Ry_{\infty} \left( \frac{1}{z_2^2} - \frac{1}{z_3^2} \right),$$

where $Ry_{\infty}$, the Rydberg constant for infinite nuclear mass, is given by

$$Ry_{\infty} = \frac{1}{8\zeta^2} \frac{me^4}{\hbar^2 c}.$$

But for real atoms (such as H for hydrogen, or D for deuterium) we need $Ry_H$ and $Ry_D$, instead, in which the electron mass $m$ is replaced by the ‘reduced mass’ $\mu_H = m \left(1 + m/M_H\right)^{-1}$ for hydrogen, or $\mu_D = m \left(1 + m/M_D\right)^{-1}$ for deuterium. This predicts, for any given transition in deuterium (vs. in hydrogen),

$$\frac{f_H - f_D}{f_H} = m \left( \frac{1}{\mu_H} - \frac{1}{\mu_D} \right).$$

And it predicts a frequency shift

$$\frac{f_H - f_D}{f_H} \approx m \left( \frac{1}{\mu_H} - \frac{1}{\mu_D} \right).$$

Similarly this ‘normal mass shift’ for the $^{87}\text{Rb} - ^{85}\text{Rb}$ comparison would predict

$$\frac{f_{87} - f_{85}}{f_{85}} \approx m \left( \frac{1}{85 \cdot 1836} - \frac{1}{87 \cdot 1836} \right) \approx 0.000270 \times 1836 = 0.147 \times 10^{-6}.$$

Hence nuclear-mass effects can account for (0.147 ppm $\times 384$ THz) $\approx 56$ MHz of the observed isotope shift. The other 28 MHz can be attributed to the ground-state electron experiencing the effect of the different nuclear size of the $^{87}\text{Rb}$ vs. $^{85}\text{Rb}$ nuclei.

Michael Crescimanno notes the valuable ‘cognitive dissonance’ this observation can create in students’ minds. Clearly, a naive Bohr-model view of the nucleus plus electron system has both particles revolving around a common center of mass – so reduced-mass effects arise naturally. But the better-trained student might think of a rubidium $S_{1/2}$ ground-state as a spherically-symmetric, and stationary, electron cloud surrounding a nucleus – and this improved picture might suggest a non-recoiling nucleus, and the absence of a nuclear-mass effect, and hence a near-absence of an isotope shift. Yet the shift exists, and students can confirm its presence! As Galileo is reputed to have said about the earth, so your students can conclude about the Rb nucleus: ‘E pur se muove’, or in translation, ‘Nevertheless, it still moves’.

In closing, we also invite you to enlarge your view from this particular Youngstown-State example to a broader question: when will your students’ work be featured in a future newsletter from TeachSpin? Feel free to e-mail dvanbaak@teachspin.com if you have a possible contribution.

The central vertical axis still shows energy, with the scale now expanded another 125-fold relative to the previous graph. The ground-state hyperfine splittings are shown correctly to scale, with two stable Rb isotopes shown separately. On this scale, the excited-state hfs splittings are under ½ mm of height, and the isotope shift of the $D_2$ transition is under ¼ mm of height. The break in the axis stands (on this scale) for the omission of just over 1 kilometer of paper.
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Inside:

We highlight an example of faculty-student collaboration at Youngstown State University, using TeachSpin equipment to teach the physics of the ‘isotope shift’ of atomic transitions.

Career opportunity:

TeachSpin Inc. of Buffalo, New York, is actively searching for an experimental physicist with experience in instrument design, advanced laboratory instruction, and experimental physics research. TeachSpin is an internationally recognized designer, manufacturer, and marketer of advanced laboratory apparatus. We focus on a single goal: enhancing the advanced-laboratory experience for the undergraduate physics major, through the creation of modern hands-on apparatus and experiments specifically for this program.

TeachSpin has played a central role in the creation of the Advanced Laboratory Physics Association (ALPhA), the NSF-sponsored BFY conferences, and the Jonathan F. Reichert Foundation. It is seeking a physicist who shares this mission of advanced laboratory education. We are certainly interested in physicists considering a “second career” with our unusual company.

Buffalo is a recovering metropolitan city, located on Lake Erie, with a vibrant theater, classical-music, and arts community. A two-century tradition of immigration, one that continues today, has led to an outstanding collection of ethnic restaurants and cultural events. The city has a rich architectural heritage, dating from the era of the “Queen City”, then the sixth-wealthiest in the USA. An era of depopulation, now ending, has provided highly affordable housing. In short, Buffalo is a welcoming and friendly community: a great place to live.

If you would like to make a difference in experimental physics education in the world, this might just be the opportunity for you to realize that goal. Contact us at JFReichert@teachspin.com.