Electromagnetically Induced Transparency (EIT) Amplitude Noise Spectroscopy

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Abstract

Intensity noise cross-correlation of the polarization eigenstates of light emerging from an atomic vapor cell in the Hanle configuration allows one to perform high resolution spectroscopy with free-running semiconductor lasers. Such an approach has shown promise as an inexpensive, simpler approach to magnetometry and timekeeping, and as a probe of dynamics of atomic coherence in warm vapor cells. We report that varying the post-cell polarization state basis yields intensity noise spectra which more completely probe the prepared atomic state. We advance and test the hypothesis that the observed intensity noise can be explained in terms of an underlying stochastic process in lightfield amplitudes themselves. Understanding this stochastic process in the light field amplitudes themselves provides a new test of the simple atomic quantum optics model of EIT noise.
I. INTRODUCTION

Electromagnetically induced transparency (EIT) [1] is a coherent multiphoton process that combines the phenomena of optical pumping and quantum interference between transition pathways into a precision spectroscopic signal with technology application in clocks and magnetometers [2–4], as well as communication schemes [5, 6] and quantum computation [7]. The light- and field-modulated optical properties associated with EIT (and related coherent phenomenon such as coherent population trapping (CPT) and electrically induced absorption (EIA)) and its ubiquity in multilevel systems makes it ideal for testing some subtleties in the atomic quantum optics model.

One aspect of EIT phenomena that has received significant experimental and theoretical scrutiny is the intensity fluctuations and correlations of light that has propagated through coherently prepared media, and using intensity noise correlation as a spectroscopic probe [8–13]. This is an example of a more general process of phase noise (PM) to amplitude noise (AM) noise conversion in atomic systems [14, 15]. In this approach for studying EIT in atomic vapor, rather than being metrologically undesirable, noise is the signal of interest. This enables the use of (phase-)noisy lasers [16] in robust technology applications and may enjoy metrological advantages over other approaches due to the existence of a regime with good signal to noise but without power broadening [17–27].

In most previous works, the intensity cross-correlation noise statistic referred to as $g^{(2)}(0)$ (defined below) is measured between light in a particular polarization basis. In this approach from either that statistic or combined with the individual channel autocorrelation statistics one is unable to quantify the light field amplitude noise. A simple change in the usual experimental protocol includes measuring the changes in the $g^{(2)}(0)$ statistic as one reversibly alters the ellipticity of the light emerging from the vapor cell. From these data the underlying amplitude noise correlations can be uniquely determined. These amplitude noise correlators lead to a more detailed test of the atomic quantum optics model developed for more basic EIT noise spectroscopy. Due to a formal connection with atomic vector magnetometry, this approach may be technologically relevant.

Existing literature studying the polarization dependence of EIT noise spectroscopy is limited. The polarization dependence of EIT and EIA signals themselves are relatively well studied experimentally and theoretically ([28–30]) as is also EIT Noise spectroscopy in the
usual circular polarization basis. As propagation eigenstates in the longitudinal magnetic field configuration, the circular polarization basis is perhaps the easiest to conceptualize EIT noise signals. However, to develop noise spectroscopy into a more versatile spectroscopic probe or apply it to a technological end, it is valuable to have a more complete understanding of the noise imprinted on the light fields by the coherently prepared atomic ensemble.

Given that motivation, we study EIT noise spectroscopy dependence on changes in the polarization basis. Recent work using restricted sets of pre- [27] and post-cell [31] polarization basis choices could not cast their data in terms of amplitude noise due to the experimental protocol. It is clear in the preceding references that the authors understood the utility of, in particular, the post-cell polarization selection as not simply creating a linear combination of noise correlators, but algebraically independent noise correlators. Carrying this implied program forward, we use noise in a third channel in a different polarization basis in cross-correlation noise measurements to explicitly construct the underlying amplitude noise correlation statistic. From that we further quantitatively test the simple atomic quantum optics model of EIT noise spectroscopy by reconstituting the intensity noise correlators in any polarization basis.

In Section II we summarize the experimental setup and protocol used. Section III describes the simple atomic quantum-optics based theoretical model accommodating this protocol and how it allows us to frame the experimental results in terms of noise in the underlying field amplitudes. Section IV summarizes the experimental findings as a test of the amplitude noise hypothesis.

II. EXPERIMENTAL SETUP

The experimental setup is typical for the Zeeman EIT noise spectroscopy and is shown in Fig. 1. The EIT state is realized in a warm (44°C) enriched $^{87}$Rb buffer gas vapor cell (length = 8 cm), buffered with 10 Torr of Neon and Argon (5 Torr Ne + 5 Torr Ar) by a linearly polarized beam from a free-running diode laser sent through the length of the cell. The linearly polarized input field (in our experiment, horizontal) has equal components $\sigma_{+}$ and $\sigma_{-}$ (right and left circularly polarized fields) of fixed phase. This light pumps atomic coherences between degenerate ground state Zeeman sub-levels, as in Hanle EIT. The degeneracy of the Zeeman sub-levels can be split by an applied longitudinal magnetic field. The amount
of splitting $\Delta$ between the Zeeman sub-levels is determined by the strength of the applied magnetic field, and subsequently we refer to it as the two-photon detuning. To ensure that a reproducible and stable splitting $\Delta$ is only a function of current applied to the solenoid around the vapor cell, it is magnetically shielded from the surrounding environment by three nested layers of $\mu$-metal magnetic shielding. (see [22] for more experimental details.)

For the two light fields from what we call the analyzer pair behind the PBS after the quarter wave plate, previous EIT noise studies indicate that a useful [16, 17, 22] noise statistic is the degree of correlation of the intensity fluctuations captured by the normalized intensity cross-correlation statistic, $g^{(2)}(0)$:

$$g^{(2)}(0) = \frac{\langle (\delta I_a)(\delta I_b) \rangle}{\sqrt{\langle (\delta I_a)^2 \rangle \langle (\delta I_b)^2 \rangle}}.$$  

(1)

The numerator of $g^{(2)}(0)$ averages the AC part of the product of the two intensities while and the denominator normalizes the result such that perfect correlation outputs $g^{(2)}(0) = +1$ and perfect anticorrelation yields $g^{(2)}(0) = -1$.

Referring to Fig. (1), most of the light (> 90%) is passed through a quarter wave-plate (QWP) before being split by a polarizing beam splitter (PBS) into linearly horizontal and vertical polarization components of what we refer to as the analyzer pair. When the QWP is aligned at 45° to the horizontal, one can view the QWP as transforming the right and left circular polarization to linear horizontal and vertical polarizations. Thus the $\sigma_+$ and $\sigma_-$ propagation eigenstates in the coherently prepared media are converted into horizontal and vertical linear polarizations.

The laser field is generated with a 795 nm free-running diode laser which is tuned via temperature and current modulation to the $F=2 \rightarrow F'=1$ hyperfine transition of the $^{87}$Rb D1 line. The diode laser’s mean frequency is stabilized with a “loose-lock” to this hyperfine transition by an analog feedback circuit whose control signal is the saturated absorption signal from another rubidium cell. The laser’s free-running linewidth of $\sim 80$ MHz is unchanged by the loose-lock, which simply prevents long term laser frequency drift; there is no other frequency stabilization of the laser (e.g. no grating feedback - internal or external). The lock used in all the data collected and described here resulted in a modest negative (i.e. red) one-photon detuning of about 10 MHz. The large spectral bandwidth of such a “noisy” laser is desirable for EIT noise correlation studies because that is more useful spectroscopically than the small obfuscation caused by the remaining laser intensity instability.
(RIN)[32]. For the diode used throughout this experiment the measured RIN was about $\sim 0.2\%$.

After it exits the atomic vapor cell, the light is split into three different polarization components, each intensity of which is measured on identical amplified silicon photodiodes whose output voltages are simultaneously digitized by a National Instruments 9223 and recorded. As described, a small portion of the light ($\approx 8\%$) is split off before the analyzer pair’s quarter-wave plate via a glass window forming the input beam to the third channel. As described below, the simultaneous recording of all three channels (two from the analyzer pair and this third channel) is necessary for the noise amplitude determination. The window is aligned at near normal incidence to leave the polarization state of both the transmitted and reflected light nearly unchanged. The polarization state incident to the analyzer pair optics is changed less than 1% by the pick-off window (that generates the third channel beam) and all the optics prior to the analyzer pair. Similar to the analyzer pair light fields, the third channel light is passed through a QWP and a linear polarizer so that the light incident on its recording photodiode corresponds to some portion of only the $\sigma_+$ light.

The intensity of the light from each of the three photodiodes is recorded in 4mS windows at a sampling rate of 1 MHz. This is done for different two-photon detunings (generated by the current in the solenoid). The intensity noise is then computed numerically from the set of measured intensities. The amplified photodiode dark current noise power was spectrally flat and varied between 2 to 12dB below the optical field intensity noise of interest. The measured intensity noise of each polarization channel has contributions from the PM to AM process due to the EIT transition, common-mode technical noise, and uncorrelated technical noise.

The effect of any common-mode technical noise is to increase the measured intensity noise correlations between the photodiodes and this increases $g^{(2)}(0)$ from its expected value. The effect of uncorrelated technical noise in the optical fields on the other hand would decrease the measured intensity noise correlations and thus reduce $|g^{(2)}(0)|$ from its expected value. The distortions in the $g^{(2)}(0)$ statistic caused by the common-mode and uncorrelated technical noise can be significantly reduced in the experimental data by appropriate Fourier filtering. The expected normalized intensity cross correlation, $g^{(2)}(0)$ between the third channel and the $\sigma_+$ channel of the analyzer pair would thus be $+1$ for all two photon detunings, with any reduction in $g^{(2)}(0)$ due to extraneous (non-atomic) technical noise. Similarly, $g^{(2)}(0)$
between the third channel and the $\sigma_-$ channel as a function of two-photon detuning should match the $g^{(2)}(0)$ between the $\sigma_+$ and $\sigma_-$ channels. However in this case, the common-mode technical noise would increase the correlation of the noise between the channels and $g^{(2)}(0)$ would increase from $-1$ when the two photon detuning $\Delta$ is in the range where the noise in the two polarizations it anticorrelated (i.e. $500 \text{ Hz} < |\Delta| < 1 \text{ MHz}$). Fourier filtering the noise data to recover this expected behavior for the identical analyzer pair and third channel quarter wave plate settings allows both the uncorrelated and the common mode technical noise to be assayed and ultimately significantly (though not entirely) filtered out of the measured data (see also [33]).

III. THEORY MODEL

The noise statistic $g^{(2)}(0)$ is but a single aggregate two-point function of the more fundamental and complete set of amplitude noise correlations wrought by the non-linear light-atom interaction. Assuming that the experimental input beam can be modelled as a single optical mode, the intensity of each orthogonal polarization is the norm-squared of an amplitude[18]. Up to an overall undetectable phase, these two amplitudes are characterized by three real quantities, indicating that there are six real statistically independent two-point functions in the amplitudes as compared with three in the intensities, $\langle \delta I_a^2 \rangle, \langle \delta I_b^2 \rangle, \langle \delta I_a \delta I_b \rangle$).

Selecting different polarization bases post-cell by rotating the quarter wave plate before the analyzer pair, sends different linear combinations of the underlying circular polarization field amplitudes to each of the channels "a" and "b". There is no way to determine the Stokes parameters for a lightfield via a single measurement of these two intensities thus the three measured intensity noise correlators at one angle for the quarter wave plate cannot, in general, determine the noise correlators one would measure at a different orientation.

In terms of circular polarization amplitudes $A$ and $B$ the measured light intensities on the photodiodes for a $45^\circ$ orientation of the quarter wave plate will be $I_a = \sigma_+ = A^*A$ and $I_b = \sigma_- = B^*B$. Each of these amplitude is complex, for example, $A = a_1 + ia_2 (a_i$ real) and in the usual way can be thought of as a two-dimensional vector $\vec{a} = (a_1, a_2)$. The amplitudes $A$ and $B = b_1 + ib_2$ are only determined up to an overall phase, the application of which to the pair of amplitudes can be thought of geometrically as the common rotation of vector $\vec{a}$ and $\vec{b}$ about the origin by an arbitrary amount. Thus, there are really only
three physical degrees of freedom in these two amplitudes as we can, with this rotation, always take $A$ to be purely real. Then the remaining six amplitude two-point functions are $<a^2>, <b_1^2>, <b_2^2>, <ab_1><ab_2><b_1b_2>$. 

Similarly, these six amplitude noise two-point functions in this simplest version, are probes of the noise in the quantum state of the atomic vapor[26]. Thus, in as far as this simple atomic quantum optics model is a description of the process, we expect a measurement of the six amplitude noise two-point function to have complete information in that they can then be used to determine the outcome of intensity noise correlation for any choice of polarization basis. Beyond the purely academic interest in showing the completeness of this description of intensity noise in terms of amplitude noise, since the off-diagonal matrix elements depend on the local field environment, this method suggests a new simple approach to sensitive vector magnetometry using vapor cells and free-running (i.e. phase noisy) laser diodes.

We do not assume Gaussian statistics for these amplitudes components, but the six two-point functions above are sufficient (in leading order) to compute the intensity noise correlations for any post-cell beam pair. Both our experimental data itself and separately the atomic quantum optics model indicate that the three point noise amplitude functions (and higher correlators) are not significantly smaller than the two point amplitude noise correlators, but we do not address this further here. Since we do not need the higher order correlators for the leading order intensity correlations, we fix the linear combination of post cell amplitudes and compute the six independent two point functions in the amplitudes at each detuning $\Delta$ for that single choice of mixing. We then use those experimentally computed amplitude two-point functions to reconstruct the intensity noise cross-correlations for different post-cell polarization basis choices and detunings. Finally, we independently experimentally measure those intensity correlations by choosing different polarization state bases (via rotating the wave plate before the analyzer pair) and comparing those data with the amplitude noise reconstructed expectations, resulting in a rather direct semi-quantitative test of this amplitude noise hypothesis.

A rather straightforward and by now standard atomic quantum optics theoretical model quantitatively captures the behavior of EIT-derived amplitude and intensity correlations. (see, for example, Refs. [16, 22]). In the simplest atomic quantum optics model for this system, the three-level $\Lambda$ system, the emergent light’s amplitudes, in magnitude and phase, encode different off-diagonal density matrix elements. Its most basic implementation is
as a density matrix for a standard a three-level Λ-system with ground states $|1\rangle$ and $|2\rangle$, and excited state $|0\rangle$ in the presence of the two EIT fields solved in steady state. The intensity cross-correlation statistic in the optically thin cell limit is computed by using the appropriate density matrix elements of the static solution to represent the slow-evolving parts of the density matrix. These include the ground state populations and coherences. To then determine the lightfield amplitudes post cell, for fixed slow-evolving parts of the density matrix one continually updates the fast-evolving parts (the parts of the density matrix involving the excited states for example) while ensemble averaging over a flat distribution of one photon detunings (a process meant to model the laser diode’s phase noise, assumed spectrally broader than the transition’s intrinsic one-photon width). With just the intensity of the two circular polarization channels $\sigma_+$ and $\sigma_-$ it is not possible to extract the underlying amplitude noise spectra. Thus a third intensity channel is added that measures light in a fixed, independent linear combination of amplitudes that make up $\sigma_+$ and $\sigma_-$. Refer to the intensity measured in this third channel as $I_c$.

Assuming that the exciting lightfields are expressible in terms of two amplitudes, one can invert the three intensity noise autocorrelations $<\delta I_a \delta I_a>$, $<\delta I_b \delta I_b>$, $<\delta I_c \delta I_c>$ and the three noise cross correlations, $<\delta I_a \delta I_b>$, $<\delta I_b \delta I_c>$, $<\delta I_c \delta I_c>$ to determine the six independent amplitude noise correlators at each two-photon detuning. In the next section we report on the result of doing this for both the experimental data and simulation output from the atomic quantum optics model. As described, we also invert the process, using the six amplitude correlators measured in one of the polarization basis to construct the expected $g^{(2)}(0)$ cross-correlation statistic for other polarization basis choices. Together these furnish a test of the hypothesis that any intensity statistic emerging from the cell can be understood in terms of a stationary Markov process in a single pair of field amplitudes.

IV. EXPERIMENTAL RESULTS AND DISCUSSION

As a first step towards understanding amplitude noise, we compare the statistic $g^{(2)}(0)$ computed in the above theory model noise spectrum with a series of measurements in which we discretely change the post-cell quarter wave plate’s orientation and the two photon detuning at fixed total optical power. With the quarter wave plate in the usual orientation, the $g^{(2)}(0)$ contrast generally increases with power and its central feature eventually broadens.
All the data shown here was taken at an optical power and beam diameter which corresponds to an intensity just beyond the beginning of the power broadening regime ([22]). Rotating the post-cell quarter wave plate (the one before the analyzer pair) from its nominally 45° orientation with respect to the input polarization changes it from the (nominal) cell propagation eigenmode amplitudes, resulting in systematic changes in the subsequently measured intensity noise spectra and correlations in the analyzer detector pair. Typical RMS noise traces in each port of the analyzer (experiment, Fig. 2a) and $g^{(2)}(0)$ experiment and theory curves as functions of $\Delta$ shown in Fig. 2b and c. We summarize our comparison between experiment and theory for EIT noise after a polarization basis change as follows:

1) Shift: the metrologically relevant central peak in the $g^{(2)}(0)$ correlator is no longer at zero two-photon detuning. The direction and magnitude of the shift is well accounted for in the simple atomic quantum optics theory model described above and in Refs. [16, 22]. For the angular displacements of the post-cell quarter wave plate beyond its nominal 45° orientation we have graphed corresponding features of experiment and theory in Fig. 3a. Although we have not included it here, the shift of the peak with power at fixed angle is also well modelled with the simple model described earlier. Also, as expected, the sign of the asymmetry and the shift flip with the sign of the one photon detuning.

2) Asymmetry: the two-photon detunings at which the anti-correlation tends to return to correlation are no longer symmetric in that detuning. The unsigned half-widths $W_{\pm}$, the detunings at which the $g^{(2)}(0)$ crosses zero, are assembled into the asymmetry $\nu = \frac{W_+ - W_-}{W_+ + W_-}$ we plot in Fig. 3b for both experiment and theory. The theory model further indicates that the asymmetry sign should flip with the sign of the angle of the quarter wave plate orientation beyond 45° degrees, an effect that was verified in the experimental data but has not been separately quantified. The fit parameters used for this comparison between theory and experiment are the total ground state decoherence rate and the optical power, fixed by data from the 45° quarter wave plate orientation only.

Next we simultaneously collected data using the third channel in a fixed different polarization basis (a 55° rotation of it’s quarter wave plate as measured w.r.t. the input polarization). Combining these three simultaneous datasets (after filtering for numerical stability and to reduce spurious experimental noise) we numerically computed the six amplitude noise correlation two-point functions $<a^2>, <b_1^2>, <b_2^2>, <ab_1>, <ab_2>, <b_1b_2>$. There were two fit parameters used to compute these correlators; a parameter that fixed the relative
intensity ratio of between the third channel and the analyzer pair (close to 8%, as measured) and an intensity offset (a DC value) applied to the signal level of the third channel alone. We then use these to estimate the cross-correlation function in a different post-cell polarization basis, for example from rotating the wave plate in front of the analyzer pair by a fixed amount. In Fig. 4 we coplot this computed $g^{(2)}(0)$ estimate along with experimental data recorded later as a function two-photon detuning for four different rotations of the quarter wave plate. To re-iterate, we use a single simultaneous data set consisting of the analyzer pair and a third channel at a fixed different quarter wave plate polarization rotation (in the actual experiment, about 10 degrees beyond the $45^\circ$ canonical orientation for the analyzer pair) to construct an expected $g^{(2)}(0)$ measured on an analyzer pair alone but whose quarter wave plate is at any angle.

The theory as described earlier predicts the overall trends in these data. The theory reconstructions as shown in Fig. 4 clearly degrade at larger angle, as shown not only by their departures from the measured curves, but also by the fact that they become more noisy. We have verified numerically that additional uncorrelated noise tends to make data as seen in Fig. 4a,b more like Fig. 4c,d. Note that adding uncorrelated noise would move all of the parts of the $g^{(2)}(0)$ curve closer to 0. In this scenario then one expects the reconstructed zero crossings of the $g^{(2)}(0)$ curve to be unaffected, as indicated in Fig. 4. Up to questions regarding the precise cause of the reconstruction’s degradation at large angle, these data and their reflection in the theory graphs in Figs. 2-4 comprise a first test of the underlying simplicity of the EIT noise amplitude hypothesis.

We have not included theory fits to the data only to simplify the presentation and focus this present paper on question of the utility, economy and completeness of deriving the observed intensity noise structure of the light emerging from this coherently prepared medium in terms of amplitude noise correlators. Likewise, we did not display other RMS traces at other quarter wave plate angles (as in Fig. 2a), though they (and their simple theory counterparts) are quite similar to the figure shown.

V. CONCLUSIONS AND ACKNOWLEDGEMENTS

In conclusion, we tested an approach to understanding of the intensity noise spectra from an EIT system as a Markov process in the two light field amplitudes. The Markov process
itself, the result of the random phase noise of the laser diode, is incorporated into a simple atomic quantum optics model in which one randomly samples a static ensemble of laser detunings. The simple consequence and utility of the EIT amplitude noise hypothesis is that given measurements at one angle of the quarter wave plate one can compute intensity correlations in any other polarization basis or field combination. Since vector magnetometers rely on detecting the other components of the magnetic field through, for example, changes in the ellipticity of the lightfields, the test performed here illuminates a new way of using EIT noise protocols in vector atomic vapor magnetometry, indicating potential utility and robustness of using noise spectroscopy in device applications such as CPT atomic clocks [34] and magnetometers [35].

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FIG. 1. (color online) (a) Experimental setup and (b) its schematic along with (c) the 3-level diagram of the minimal atomic quantum optics model of EIT.
FIG. 2. (color online) (a) Experimental RMS noise in each of the analyzer ports as a function of the two-photon detuning with the post-cell phase plate held at 45° ($\sigma_+$ in Red ‘+’s, and $\sigma_-$ in green ‘x’s). (b) The cross-correlation statistic, $g^{(2)}(0)$ versus detuning measured (points) and theory (line) for the cross-correlation statistic for the post-cell phase plate held at 45° and (c) Experiment (points) and theory (line) again for the post-cell quarter wave plate held at 60°. The lack of smoothness of the theory curves is due to finite sample size for each detuning (here chosen to match that of the experimental data).
FIG. 3. (color online) (a) Shift of the $g^{(2)}(0)$ maxima, experiment (red "+" s') and theory (green line) with wave plate angle beyond 45° degrees. Also shown is the FWHM, experiment (blue stars) and theory (cyan line). (b) The asymmetry $\nu_3$ as a function of the wave plate angle beyond 45° degrees.
FIG. 4. (color online) Measured $g^{(2)}(0)$ EIT intensity correlation curves (in green, 'x's) and their reconstruction using amplitude noise analysis (in red, as "+"s), as a function of the two-photon detuning at different post cell quarter wave plate orientations. All angles are displacements from the nominal 45° orientation. (a) 5 degrees additional rotation, (b) rotated an additional 10 degrees, (c) additional 15 degrees, (d) additional 20 degrees rotation.