Capstone

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Introduction to Quantum Mechanics

- States in Hilbert Space

$$[\hat{x}, \hat{p}] = i\hbar$$

- Superposition of Basis States

$$|\phi\rangle = a_1 |1\rangle + ... + a_n |n\rangle$$

- Schrödinger Equation, analog to Newton

$$i\hbar\partial_t |\phi\rangle = \hat{H} |\phi\rangle$$

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Harmonic Oscillator

- Consider the following Hamiltonian...

$$H = \frac{p^2}{2m} + \frac{1}{2}\hbar\omega^2 x^2$$

- Position Basis Solution
- Weyl Algebraic Solution via Ladder Operators

$$a = \sqrt{\frac{m\omega}{2\hbar}x + i\sqrt{\frac{1}{2m\omega\hbar}}p}$$

$$a^\dagger = \sqrt{\frac{m\omega}{2\hbar}x - i\sqrt{\frac{1}{2m\omega\hbar}}p}$$

$$E_n = (n + \frac{1}{2}\hbar\omega$$

Multi-particle Systems

- Treat Multple Particle as Direct Product of Single Particle Systems
- Interchange of Identical Particles descibed by same state (up to scaling)
- Pick up phase depending on type of particle: Bosons $(+1)$/ Fermions (-1)
- Negative Sign leads to Pauli Exclusion Principle

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Lieb-Liniger Model

- 1-D Interacting Boson Model with Periodic B.C.

$$H = -\sum_{j=1}^{N} \frac{\partial^2}{{}^{2}_{j}} + 2c \sum_{1<=i<j<=N} \delta(x_i - x_j)$$

$$\psi(x_1, ..., x_N) = \sum_{P} a(P) exp(i \sum_{j=1}^{N} k_{p_j} x_j)$$

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- Rb-87 in Light Trap (Harmonic)
- Fits 1-D interacting bose model, acts like fermi gas at large coupling

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- Flow Monotonically Increasing
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Zuo-Gao Approach

- Model the system of bosons using Quartic Interaction

$$\mathcal{L} = tr(\partial_t M \partial_t M^\dagger) + tr(MM^\dagger) + g_4 tr(MM^\dagger MM^\dagger)$$

- Taking the Large-N Limit,

$$N^2\epsilon(g_4) = N\epsilon_F - \int \frac{dx}{3\pi}(2\epsilon_F - x^2 - 2g_4x^4)\theta(2\epsilon_F - x^2 - 2g_4x^4))$$

$$N = \int \frac{dx}{\pi}(2\epsilon_F - x^2 - 2g_4x^4)^{\frac{1}{2}}\theta(2\epsilon_F - x^2 - 2g_4x^4)$$

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Zuo-Gao Approach

- Deal with Collective Field for Convenience in a general system.

$$\phi(x) = \int \frac{dk}{2\pi} e^{ikx} tr(e^{-ikM}) = \sum \delta(x_i - x_j)$$

- Effectively a "density", which can describe our system. As ϕ is a function of x, we can Fourier transform.
- Consider the general Hamiltonian

$$H = \frac{1}{2}\sum p_i^2 + \frac{1}{2}\sum v(x_i, x_j) + \sum V(x_i)$$

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Coupling and Chemical Potential

- Taking the derivative w.r.t. $\rho(u)$ leads to

$$\frac{\pi^2}{2}\rho^2(u) + \alpha\rho(u) = E - \frac{1}{2}\omega^2 u^2$$

- Simple application of Quadratic Formula yields

$$\rho(x) = \frac{-a + \sqrt{a^2 + 2\pi^2 E - x^2}}{\pi^2}$$

- However, from QM, we have a normalization condition on $\rho(u)$, leading to

$$\frac{\pi}{2} = -\frac{\frac{\alpha}{\sqrt{\omega}}\sqrt{\frac{E_\alpha}{\omega}}}{\sqrt{2}\pi} + \left(\frac{\frac{\alpha^2}{\omega}}{2\pi^2} + \frac{E_\alpha}{\omega}\right) asin\left(\sqrt{\frac{E_\alpha}{\frac{\alpha^2}{2\pi^2\omega} + \frac{E_\alpha}{\omega}}}\right)$$

- Numerically Solve for Energy

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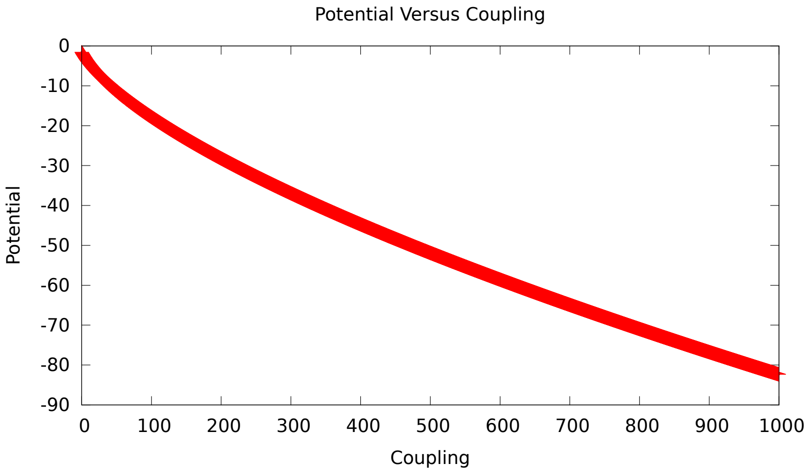
Chemical Potential

- Chemical Potential

$$V_{eff} = N^2 \int du \Big(\frac{\pi^2}{6} \rho(u)^3 - (E_\alpha - \frac{1}{2}\omega^2 u^2)\rho(u) + \frac{\alpha}{2}\rho(u)^2 \Big)$$

- Solved Analytically

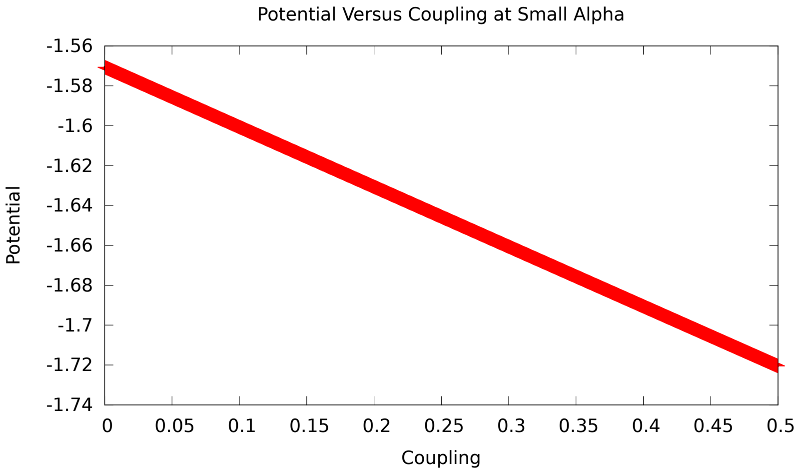
Chemical Potential versus Coupling



Small Coupling Limit

- ???

Chemical Potential versus Coupling (Small Coupling)



Large Coupling Limit

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Chemical Potential versus Coupling (Large Coupling)

LargeAlphaRange.pdf

Conclusion

- Yep

Bibliography

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Strauss, W. , *Partial Differential Equations: An Introduction*, Wiley, Danvers, MA, 2008.