Multi-horizon Black Hole deSitter Spacetimes

Dipesh Bhandari (YSU)

Dr. Mike Crescimanno (YSU)

Solutions to Einstein Field Equations

$$ds^2 = \left(1 - \frac{2m}{r}\right) dt^2 - \frac{dr^2}{1 - \frac{2m}{r}} - r^2 d\theta^2 - r^2 \sin^2\theta \, d\varphi^2$$

Schwarzschild metric

$$N(r) = 1 - \frac{2m}{r} - \frac{\Lambda r^2}{3}$$

de Sitter

$$ds^2 = -\left(1 - \frac{2M}{r} - r^2\right) dt^2 + \frac{d\xi^2}{1 - \frac{2M}{r} - r^2} + r^2 d\Omega.$$

De-Chang Dai ("squiggle" coordinates)

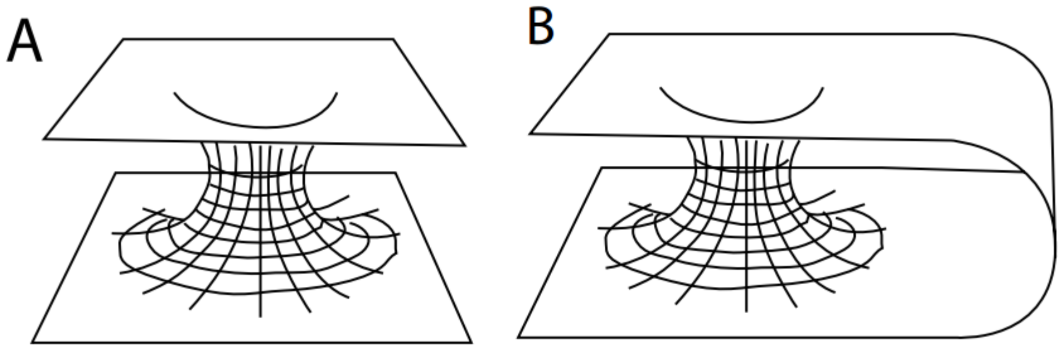
- **closed de Siter spacetime**
- **making the cosmo-horizon "accessible"**
- **connecting two halves of the spacetime**

$$N(r) = 1 - \frac{2m}{r} + \frac{q^2}{r^2} - \frac{\Lambda r^2}{3}$$

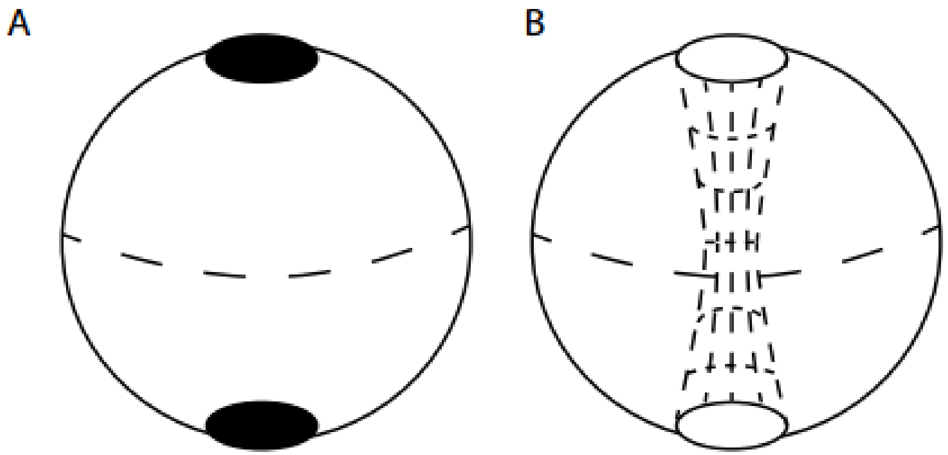
Reissner-Nordström (notice charge)

More on Dai's Wormhole Solution

$$ds^2 = -(1 - \frac{2M}{r} - r^2)dt^2 + \frac{d\xi^2}{1 - \frac{2M}{r} - r^2} + r^2 d\Omega.$$



A – Einstein and Rosen's original solution (two universes).
B –Two universes connected at infinity (looks like a single universe, black holes are infinitely far away).



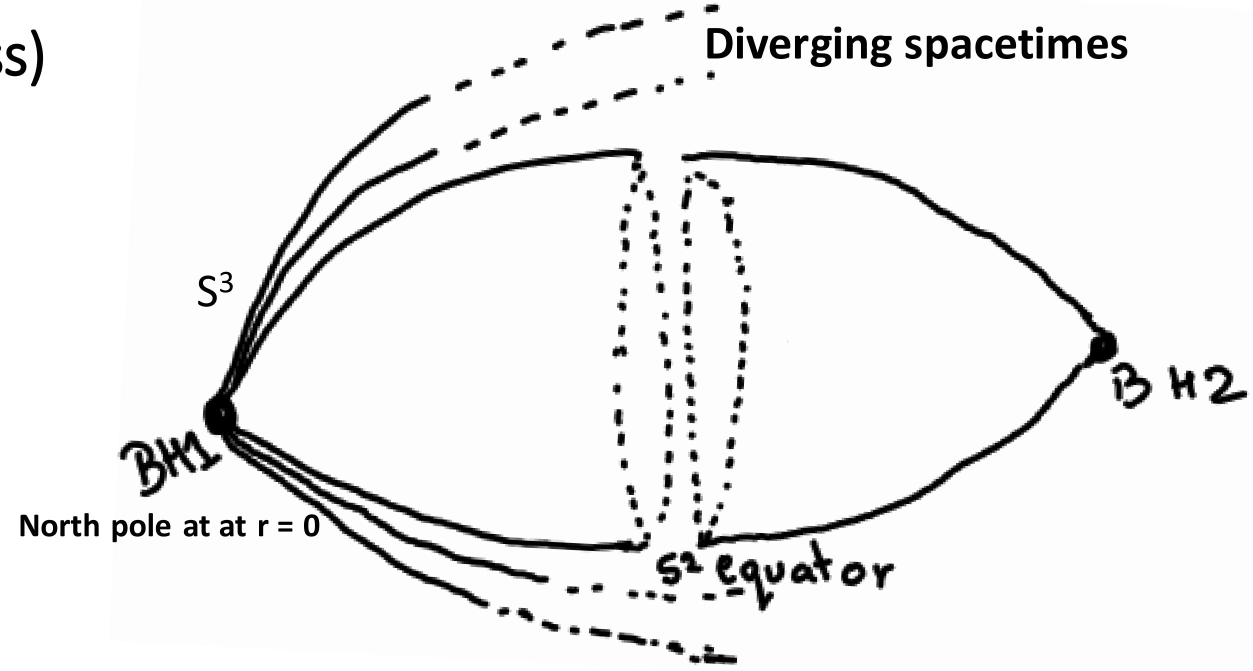
- Schwarzschild black holes placed at antipodes of closed de Sitter space (+ve cosmological constant)
- No net force at the equator

$$\partial_r g_{tt} = 0$$

Much more on Dai's solution... (process)

$$ds^2 = -(1 - \frac{2M}{r} - r^2)dt^2 + \frac{dr^2}{1 - \frac{2M}{r} - r^2} + r^2 d\Omega.$$

- Solution for one half (deSitter, vacuum, Static)



Equator at $r_0 = M^{1/3}$ using the condition $\partial_r g_{tt} = 0$
The other half's description using a coordinate transformation using λ

$$(r - 2M - r^3) = a^2 \sin^2 \lambda$$

0, π/2, π --->> BH1, equator, BH2

No coherent description of the equator because the dr component = 0 still --->> $r = r_0 - |\xi|$

$$ds^2 = -(1 - \frac{2M}{r} - r^2)dt^2 + \frac{d\xi^2}{1 - \frac{2M}{r} - r^2} + r^2 d\Omega.$$

Holes at
$\xi = \pm(r_0 - r_h)$

Components of Stress-Energy Tensor (ξ coordinates)

$$\bar{T}^t_t|_{\xi=0} = \frac{-8\delta(\xi)}{rB}$$

$$\bar{T}^\lambda_\lambda|_{\xi=0} = \frac{4\delta(\xi)}{rB}$$

$$\bar{T}^\theta_\theta|_{\xi=0} = G^\theta_\theta|_{\xi=0} = \frac{2\delta(\xi)}{rB}$$

<-- $\partial^2_\xi r = -2\delta(\xi)$

Light-like matter in the equatorial S2 shell.

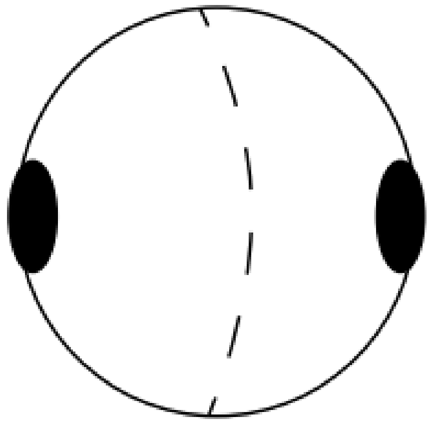
We attempt to connect the two spacetimes <u>without the light like matter shell</u> in the equator.

Using $D_\mu T_{\mu\nu} = 0$ and components of Ricci Tensor for our Ansatz
$ds^2 = A(r)dt^2 - B(r)dr^2 - r^2 d\Omega^2$

$$T_{\mu\nu} = \ diag \ \left(t_t, -t_r - \mathrm{P}r^2, -\mathrm{P}r^2 \, s_\theta^2\right)$$

$$\frac{1}{rB}\left(\frac{A'}{A} + \frac{B'}{B}\right) = \frac{\kappa t(r)}{A} \qquad \frac{rA'}{A} = B(1 - \Lambda r^2) - 1 \qquad ***$$

1) Weak Energy Condition - **t(r)>0** (exterior to all BH horizons).
2) Strong energy condition – **$T = g^{\mu\nu} T_{\mu\nu} \geq 0$** (exterior to all BH horizons).
3) Global Single time – **metric signature (+---)** everywhere.
4) Horizon regularity – Where A vanishes, **A' does not.**
5) Matter regularity - invariants computed using $T_{\mu\nu}$ **finite** (exterior to all BH horizons).



GOAL: Find B(r) leading to solutions of ***
- satisfy all the above conditions
- contain two blackhole horizons antipodally.

$A(r) = 1 - \Lambda r^2/3 - M/r = 1/B$, $\kappa t(r) = 0$, (empty deSitter space) but this has only one blackhole and one cosmological horizon. [possible **uninteresting** solution]

Finding Out The Boundary Conditions For Our Solutions

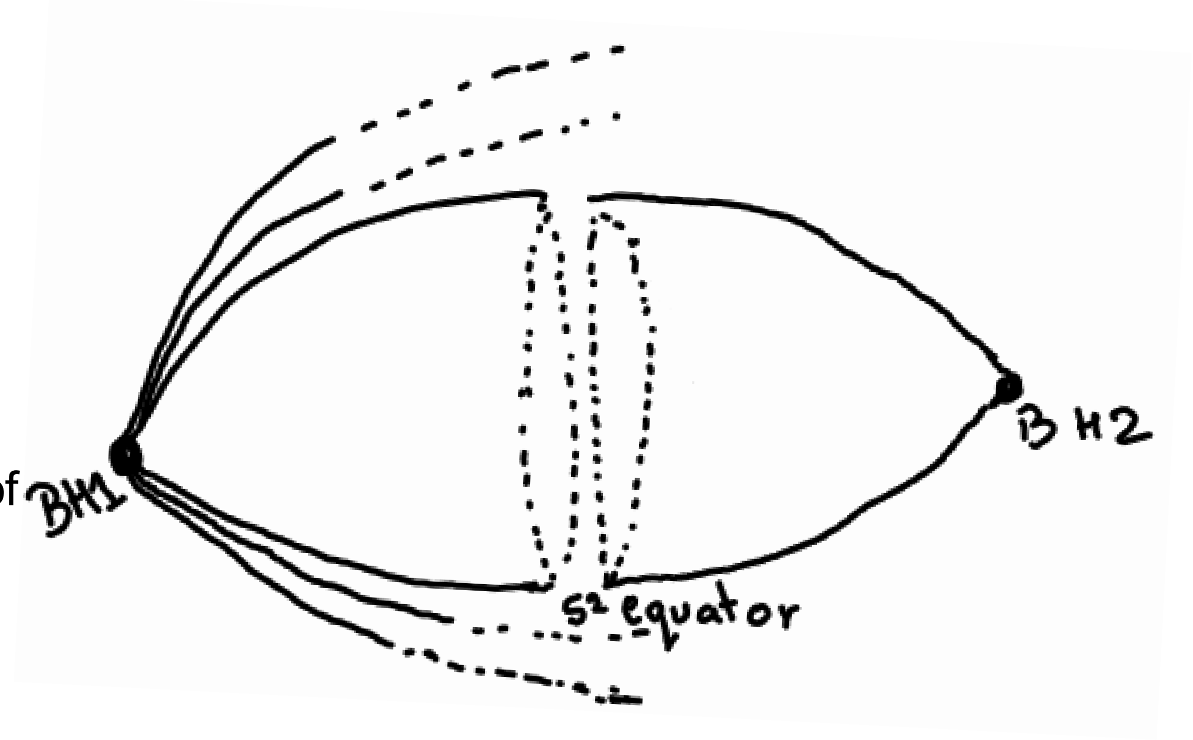
Using matching conditions of the two spacetime slices.

S^2 slice of the same geometric nature gives $r = \tilde{r}.$

Tilde --> second half of the spacetime

Let $r = \tilde{r} = r^*$ ----> S^2

Particle released from rest there must accelerate the same (as seen in its local co-ordinates) on either side of the bounding S2 .



accelerations (using radial components of Geodesic equation) leads us to $B\mathrm{d}r^2 = \tilde{B}\mathrm{d}\tilde{r}^2,$

(which is because of ds^2 invariance), yields...

$$\frac{A'}{A\sqrt{B}} = -\frac{\tilde{A}'}{\tilde{A}\sqrt{\tilde{B}}}$$ **!!!**

Angular geodesic equation

[Same solutions on either side of boundary like before]

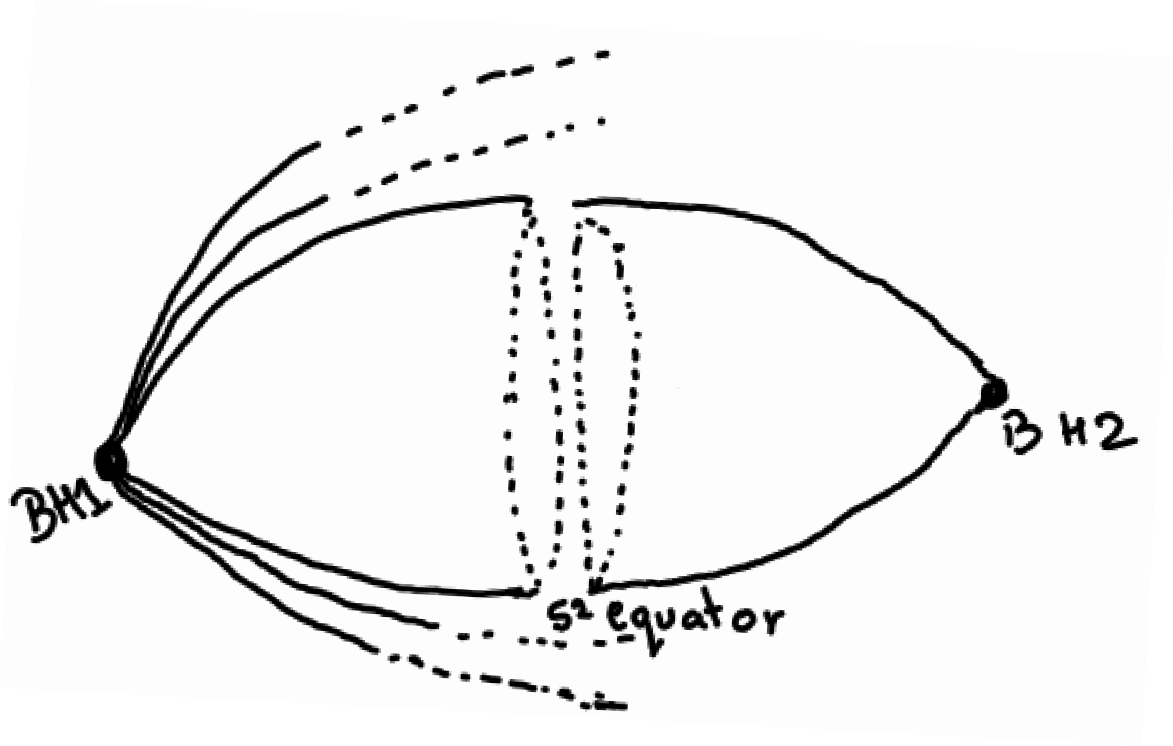
$u_T = (\alpha, 0, 0, \beta)$ in the $\theta = \pi/2$ plane.

$\beta = d\phi/ds = -\tilde{\beta}$.

Timelike condition $u^T u_T = 1$ for both halves indicates that

$$A\alpha^2 = \tilde{A}\tilde{\alpha}^2.$$

Non-trivial component of geodesic equation -->
$\beta^2 = A'\alpha^2/(2r)$



$$A'/A = \tilde{A}'/\tilde{A}.$$

$A' = 0 = \tilde{A}'$ at r* (using the !!! equation)

B = B′ at that r*

$t/A = \tilde{t}/\tilde{A}$ at $r = r^*$. (Trace of T single valued for two sides)

$B' = \tilde{B}'$ at r^*. (using !!! Again)

Goals:

- **Write a code to find solutions (A and B in our ansatz)**

$$ds^2=A(r)dt^2-B(r)dr^2-r^2d\Omega^2$$

Also, impose the conditions below!

1) The solution also has A′= 0 at the same r*.
2) the B are equal at r*.
3) The B′ are equal at r*   (r* is the location of the connecting S^2 slice)

Digression: What made us think that we could find such solutions where we can connect the two spacetimes without the matter shell?

Consider dS blackhole solution (no mass distribution) in which A= 1/B globally

Extremal limit in which the blackhole horizon and the cosmological horizon coalesce.

A has double root, $A = A' = 0$ $B \to \infty$,

Geometrically an infinitely long tube is developed at r*

Two of the tubes can be joined with arbitrarily little matter.

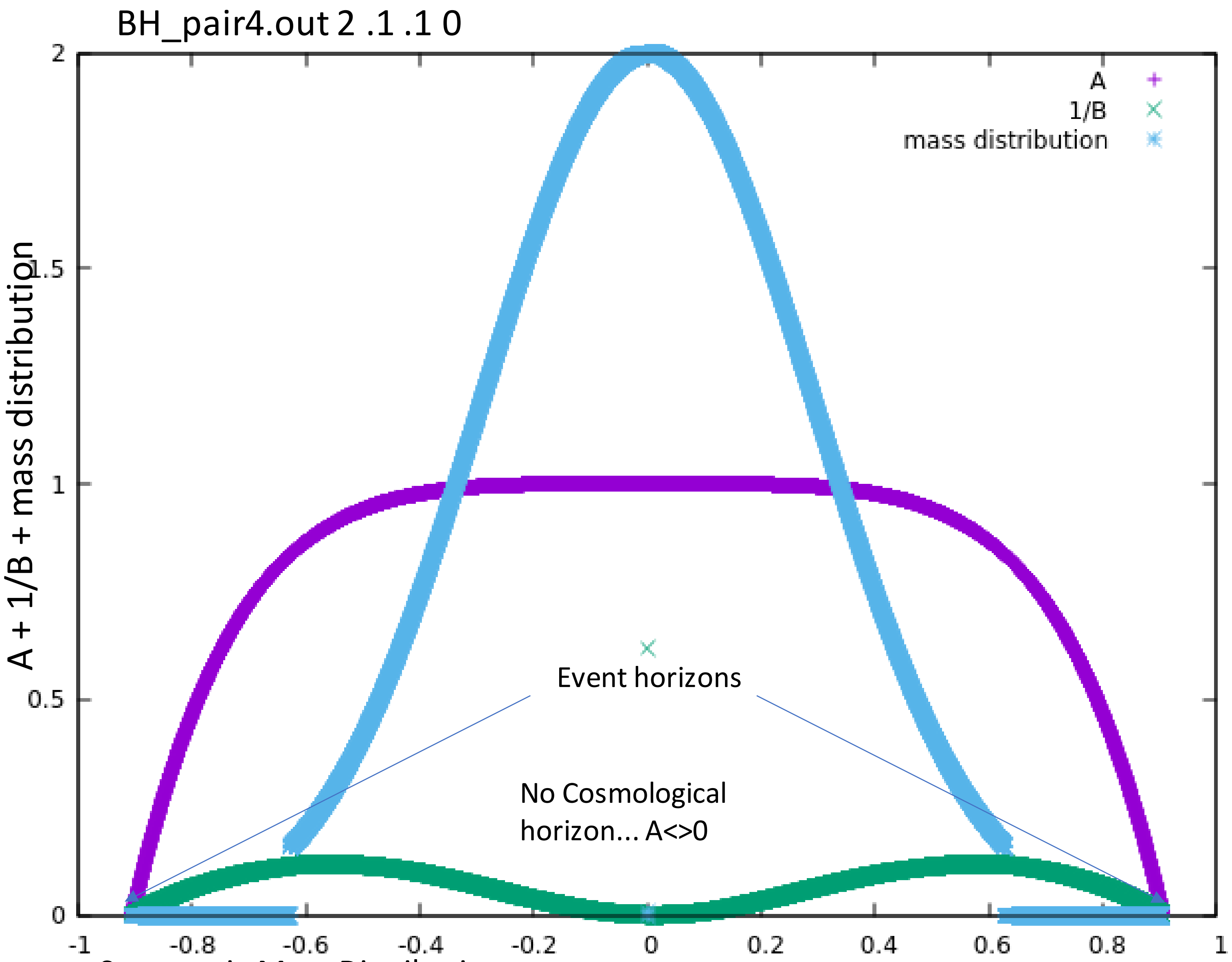
Possibility of finding a global solution of the dS+blackhole+matter system without any singular mass distribution at all.

Let B have a simple pole at $r^*= 1$.

With a simple pole in B as $A' \to 0$ at $r^*= 1$, leads to a harmless co-ordinate singularity there.

This is only possible if $t/A \neq 0$,and that, t/A is well defined there.
Brief calculation indicates that **$t/A \to 2$ at $r \to 1$.**

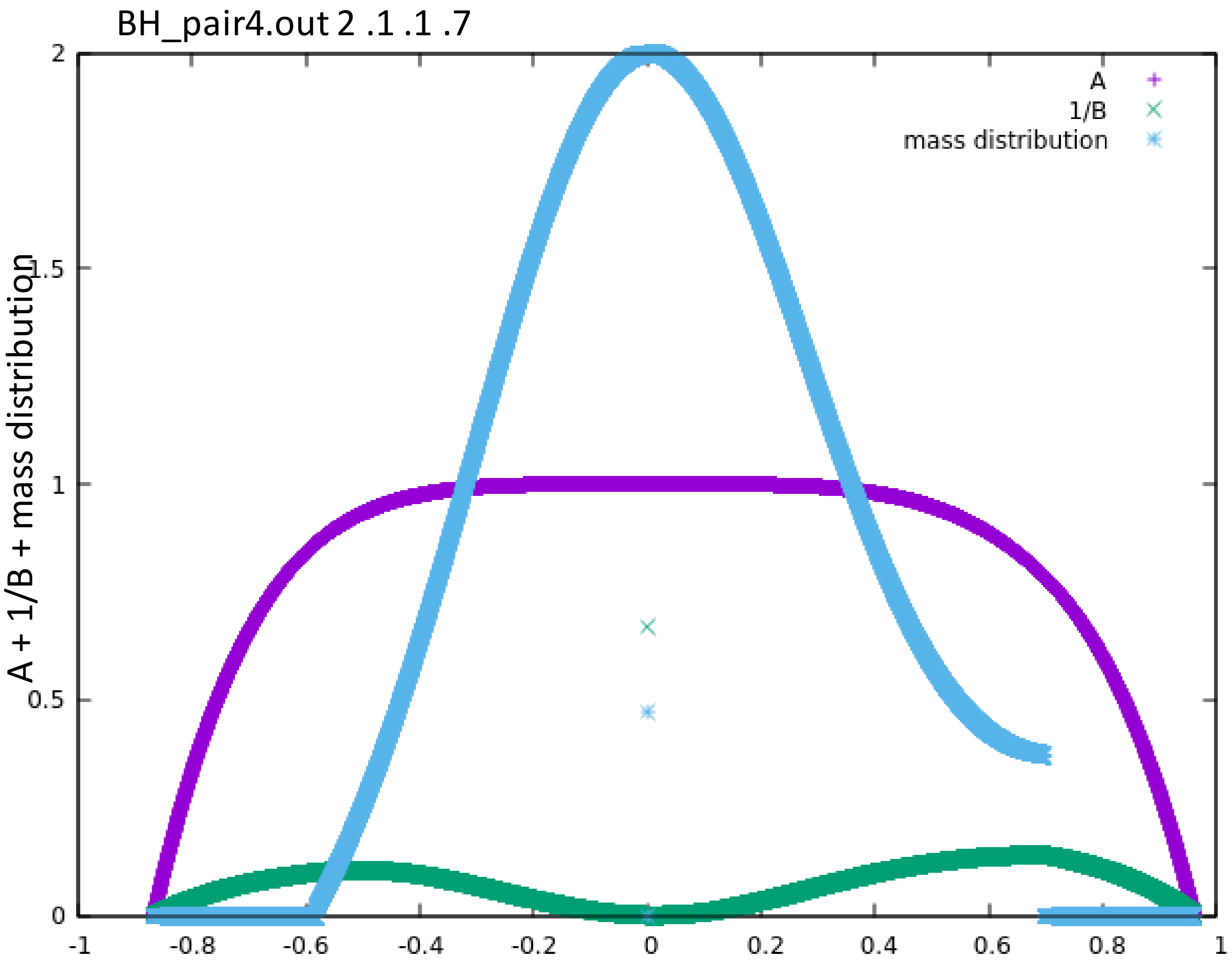
A few millennia later…



Symmetric Mass Distribution

A' = 0 at the equator

1/B = 0 at event horizons and NO cosmological horizon



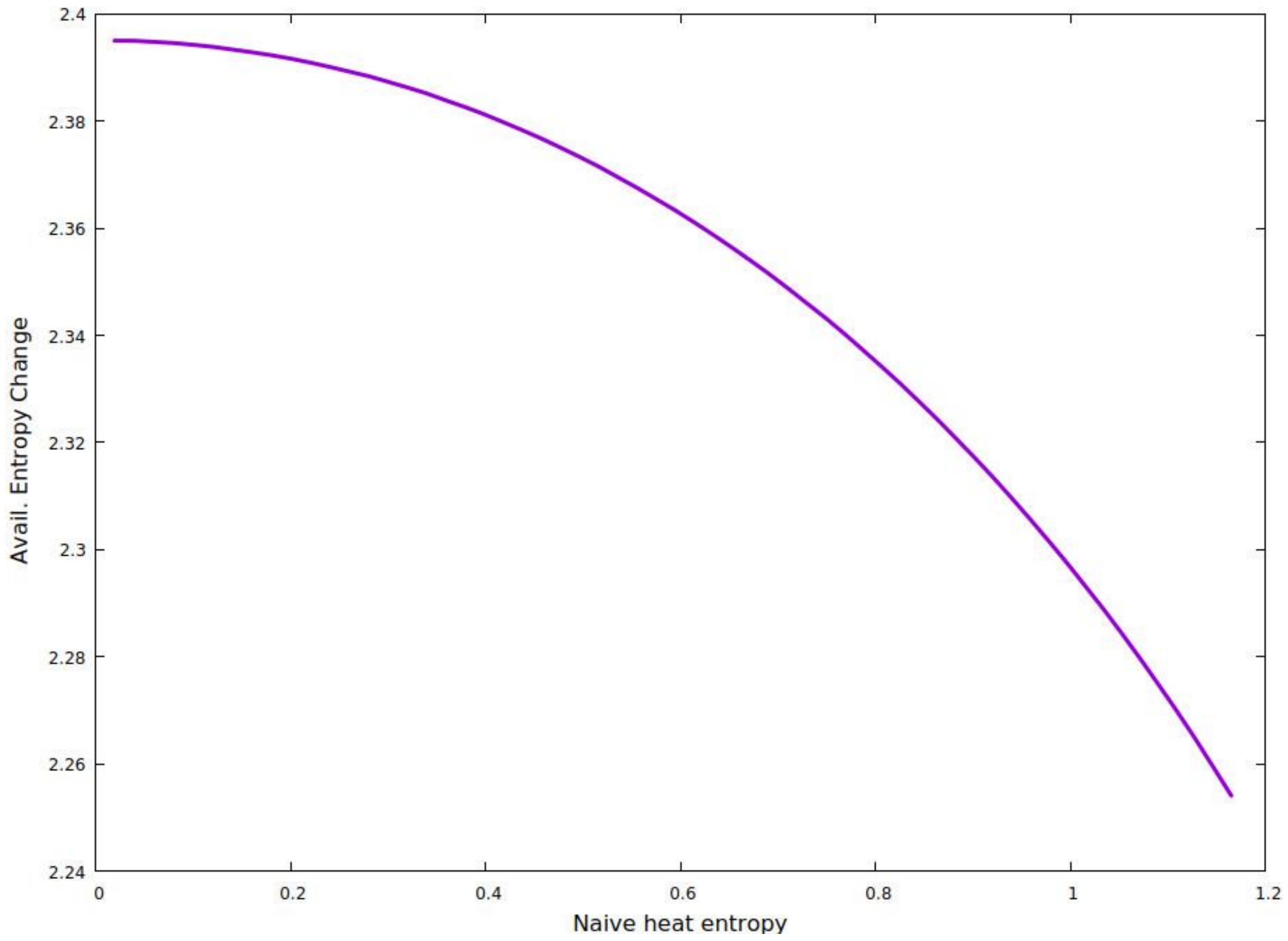
Asymmetric Mass Distribution

A' = 0 at the equator

1/B = 0 at event horizons and NO cosmological horizon

Peeking into the Thermodynamics

- We've found solutions that are interesting from the point of view of studying thermodynamic effects in semiclassical gravity in a finite volume, finite size spacetime.

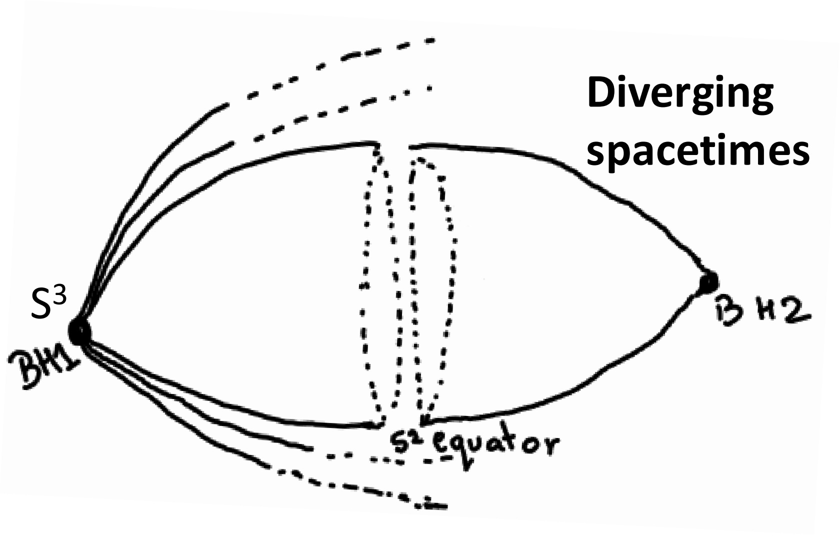


Complicating things with charges...

$$N(r) = 1 - \frac{2m}{r} + \frac{q^2}{r^2} - \frac{\Lambda r^2}{3}$$

Reissner-Nordström (R-N)

(With Λ = 1)



<u>CONDITION A:</u> Where the sign of the metric changes.

$$g_{tt} = 0$$

$$r^4 - r^2 + 2Mr - q^2 = 0$$

$$x^3 + \left(-\tfrac{1}{3} + 4Q^2\right)x + \left(-\tfrac{16}{27} + \tfrac{2}{3}\left(1 + 4Q^2\right) - 4M^2\right) = 0$$

$$x = \frac{P}{3}\left\{-\frac{L}{2} \pm \sqrt{\left(\frac{L}{2}\right)^2 + \left(\frac{P}{3}\right)^3}\right\}^{-\frac{1}{3}} - \left\{-\frac{L}{2} \pm \sqrt{\left(\frac{L}{2}\right)^2 + \left(\frac{P}{3}\right)^3}\right\}^{\frac{1}{3}}$$ where

$P = \left(-\frac{1}{3} + 4Q^2\right)$ and $L = \left(-\frac{16}{27} + \frac{2}{3}\left(1 + 4Q^2\right) - 4M^2\right)$

TRANSFORMATIONS USED: $u = x + 2/3$ AND $a^2 = u$

where a is one of the coefficients of the original equation $r^4 - r^2 + 2Mr - q^2 = 0 = (r^2 + ar + b)(r^2 + cr + d)$

The discriminant!

$$\left(\frac{L}{2}\right)^2 + \left(\frac{P}{3}\right)^3 > 0$$

$$0 < M < \left(-\frac{4}{27} + \frac{1}{6}\left(1+4Q^2\right) - \frac{1}{2}\left(\frac{1}{9} - \frac{4Q^2}{3}\right)^{\frac{3}{2}}\right)^{\frac{1}{2}}$$

which is true iff $-\sqrt{\frac{1}{12}} < Q < \sqrt{\frac{1}{12}}$

Similarly…

Condition B: The force equilibrium condition: $\partial_r g_{tt} = 0$

Same story with quartics:
BUT simpler and the discriminant gives us the condition:

$$0 < M < \frac{4Q^{\frac{3}{2}}}{3^{\frac{3}{4}}}$$