Physics 3704: Experiment 6

A. Electron diffraction

B. The Charge of an Electron

A. Electron diffraction

The first person to suggest that an electron would have an associated wavelength was Louis de Broglie in his doctoral thesis in 1924[1]. Previous experiments on the setting on the scattering of slow electrons[2] had apparently revealed some of the wave behavior of electrons but weren't interpreted in this light until after de Broglie's announcement[3]. The hypothesis made by de Broglie was that the wavelength of an electron was related to its mass and speed by

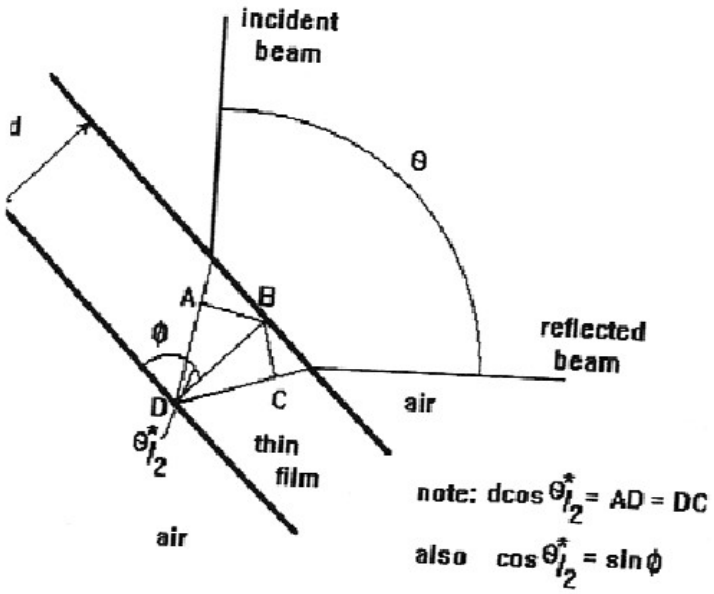
$$\lambda = \frac{h}{m_0 v} \qquad (5.1)$$

A more general statement, one that holds true even when relativistic speeds are used for the electrons, is

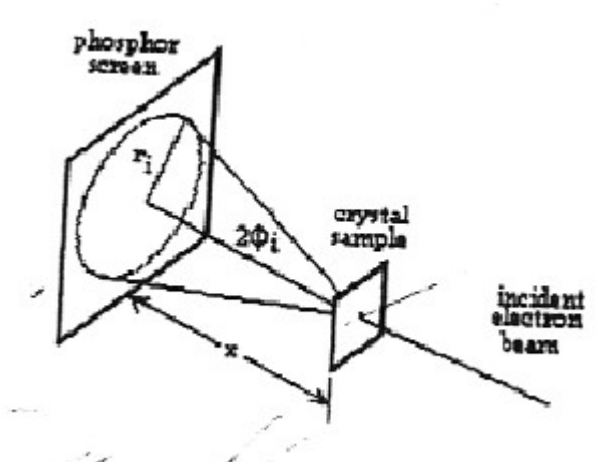
$$\lambda = \frac{h}{p} \qquad (5.2)$$

where $p = \gamma m_0 v$, the relativistic momentum.

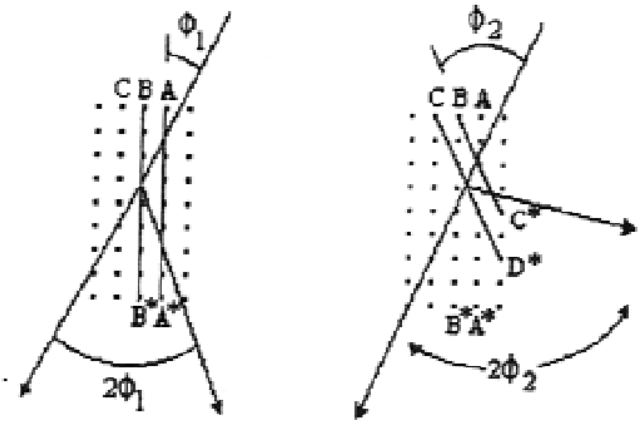
To help understand the behavior of electrons diffracted from crystals, an analogy can be made between the familiar analysis of thin film reflection from geometric optics and the scattering of an electron from a crystal. Either of these cases can be used to derive the Bragg condition $(2 d sin\theta = n\lambda)$, where in the case of crystal diffraction, n is an integer (not the index of refraction). Examine the case of thin film reflection shown in the diagram on the next page, notice that if the difference in path length (AD+DC) between the incident wave font AB and the emerging reflected wave front BC is an integer number of wavelengths (in the medium), the constructive interference will occur (ignoring phase change upon reflection), and a maximum in reflected light will be seen at the angle θ with respect to the incident beam. You may also notice that this is NOT the typical treatment in an introductory text where the incident angle is assumed to be near-normal. Also, in the index of refraction of the thin film is very near the index of surrounding medium $\theta^{¿} = \theta$ .The case of diffraction from a crystal does not have the complications of phase changes upon reflection and the refraction upon entry into the medium that is seen in thin film reflection, but it does have other complications due to the multiple symmetry plans of the crystal.


incident
beam
d
θ
A
B
ϕ
reflected
beam
D
C
air
$\theta^*_{l_2}$
thin
film
air
note: $d\cos\theta^*_{l_2} = AD = DC$
also $\cos\theta^*_{l_2} = \sin\phi$

We want to do an experiment where the electron beam passes through a crystal of aluminum. In our situation, we must consider "grazing" angles. So instead of looking at reflections, we will look at electron transmission through a crystal sample. Our experimental geometry is shown in the figure to the right. The distance (x) between the crystal sample and the phosphor screen is 18.2 cm. we expect several circular patterns of radii r_i described by cones with angles $2\phi_i$. Often, the distance between the screen and sample is much larger than the radical distance r_i. The approximation is then made where $\tan 2\phi_i \approx \sin 2\phi_i \approx 2\phi_i = 2r_i/x$.



The diagram to the right shows (in two dimensions) what can happen when an electron beam deflects from a 2-dimensional crystal plane at a grazing angle. The parallel planes of atoms in crystal AA* and BB* will deflect the incident beam by 2, and the parallel planes of atoms in the crystal BC* and CD* will deflect the incident beam by $2\phi_2$. In 3 dimensions, these deflections describe cones. The actual situation in aluminum which is referred to as face-centered-cubic (FCC). You can visualize the FCC structure of aluminum atoms at its corners and aluminum atoms at its corners and aluminum atoms at the center of each face. The distance between reflection planes for the FCC structure is often described by Miller Indicates h,k,l, as



$$d = \frac{n a^{\prime}}{\sqrt{h^2+k^2+l^2}} \tag{5.3}$$

where a* is the lattice constant (the distance between planes of aluminum atoms). You can think of the Miller Indicates as a way of characterizing the geometry of the various reflecting planes of the crystal in equation form.

Using the Bragg relation and the small angle approximation, you should verify that

$$\lambda = \frac{r a^{\prime}}{x\sqrt{h^2+k^2+l^2}} \tag{5.4}$$

If one obtains a* from other sources (like x-ray diffraction experiments) and measures r and x, all that is left to do is find proper values for h, k, and l to get . It turns out the smaller proper values for h, k, and l from the allowed FCC reflections in aluminum are

h	k	l
1	1	1
2	0	0
2	2	0
3	1	1
2	2	2

4 0 0

The goal of this experiment is to measure as a function of electron speed using the symmetry of the crystal as a target.

The Sargent-Welch Electron Diffraction Apparatus has four target areas mounted within the electron tube. You can see these target areas if you shine a flashlight into the side window and look into the tube between the vertical bars. These target areas look like window screen material coated with shiny particles. Three of the target areas have polycrystalline aluminum on them, and one area has pyrolytic graphite on it. We will use the aluminum targets only. When the tube is in operation, the various areas can be accessed with the electron beam by using the vertical and horizontal steering controls on the front panel.

Turn on the apparatus with care. Make certain that the accelerating voltage knob is turned all the way down (counter-clockwise) before lifting the AC switch to turn it on. Let the electron gun inside warm up for 10 minutes. (You should see it glow in the back of the tube.) Turn the high voltage up to 4kV and adjust the focus, intensity, vertical, and horizontal knobs so that they are all pointed vertically upward. Once you get a visible spot on the screen, you can adjust the controls as needed. If no spot appears, turn up the intensity knob slightly. Try to operate the device at the lowest possible intensity setting to help maintain the lifetime of the electron gun within the tube. If you turn off the room lights, the radii of the diffracted electrons will be more easily viewed.

Try to measure the 5 smallest radii for the three aluminum target areas at 5 different electron speeds. (Use 4kV, 5kV, 6kV, 7kV, and 8kV for V_{acc}.) If a different person measures the radii for each target area, you can average the results and find more objective final averages. Use your average radii, the Miller Indicates, the screen-to-target distance, and the lattice constant of aluminum from x-ray data (a^*=4.05 angstroms)[4] to calculate the wavelength of the electrons at each energy. Compare your results to the value obtained using the de Broglie relationship and determine your uncertainties and discrepancies. You may want to check how much error is introduced by using a non-relativistic approach when finding p, and estimate how good the values for V_{acc}, x, and uncertainties in your results?

B. The Charge of an Electron

An Indirect Measurement of the Charge of an Electron in terms of Avagadro's Number

Purpose: As an addendum to the *e/m* experiment that you do we suggest that you also do the following, very brief experiment that will allow you to measure the charge e of an electron. Combining the results of these two experiments will thus yield also a measurement for the mass *m* of an electron.

Apparatus: A beaker, 0.2 molar $CuSO_4$ solution, copper plates (2), rubber bands, and a DC power supply.

Discussion: The idea of this experiment is very simple. We use an electrolytic cell and a DC power supply to deposit copper onto one of the plates in the electrolytic cell. As the electro plating process ensues we record the time and the current that is flowing. (A constant current supply will ideally be provided by your lab instructor. If none is available your instructor will advise you how to proceed). An ampere is one coloumb flowing past a point in the wire per second, and so the product of the time (in seconds) we have had the current flowing in the cell and the current (in amps) is the number of coloumbs of charge that have coursed through the wires and therefore through the electrolytic cell.

$$Q = ¿ \qquad (5.5)$$

Then, by weighing the amount of copper that has been deposited we will know how many moles of electrons have been through the cell. (A mole of copper weighs 63.5 grams) Thus, using Avagadro's number (i.e. 6.02×10^{23}) we will be able to compute the charge (in Coloumbs) per electron.

Before outlining the procedure let us describe the operation of the electrolytic cell. In essence, an electrolytic cell consists of two conducting plates placed in container holding a solution (draw a figure in your report). The solution in this experiment is simply water in which some copper sulfate has been dissolved. When a "salt" such as $CuSO_4$ is dissolved in water it disassociates into ions, in this case, Cu^{+2} and SO_4^{-2} ions. As "solvated ions" (that is, ions covered with water molecules) they are essentially non-interacting and move rather freely throughout the liquid.

When a D.C. voltage V is put across the cell there arises an electric field $\vec{E} = V/d$ between the electrodes (in this case copper two plates). The ions respond to the force $\vec{F} = q\vec{E}$ and drift in this applied electric field towards the plate of the sign opposite their own. This motion of charges inside the liquid constitutes a current equal in magnitude to that which is flowing in the circuit. This is due to the fact that when the ions, say that the Cu^{+2} ions get to the negative plate they steal two electrons from that plate and then lay as reduced (metallic) copper, Cu, on or near that electrode. Two electrons flow in from the rest of the circuit to replace those two that were absorbed by the Cu ion.

A similar process occurs at the other electrode. The SO_4^{-2} ions collect and donate two electrons to the plate. The process, chemical reactions occur at the plate which cause gas (SO_2) to be evolved from that electrode and cause several other processes at the plate occur which cause that plate to erode, that is, causes some of copper on that plate to go into solution.

The basic point is that for each Cu atom that is reduces on the plate, two electrons will have flowed through your circuit.

Safety Precautions

1) NEVER TOUCH THE ELECTRODES WITH WET HANDS WHILE THE D.C. VOLTAGE IS ON. Just shut off the call and proceed. The entire experiment happens at pretty low voltage BUT even at low voltage, if you are a good enough conductor, you may indeed get a nasty shock.

2) DON'T SNIFF THE (rather small amount of) GAS THAT IS EVOLVED BY THE PLATES. Please be sure that you do this in a reasonably well ventilated room. Open the window. It's a small amount of gas and so it shouldn't be any problem. BUT don't consciously try to inhale it!
3) TAKE CARE IN HANDLING THE BEAKER AND THE SOLUTION. Ordinary common sense precaution is in order here.

Procedure:

Begin by cleaning and drying the copper electrodes that are provided. After wiping them with a cloth it is best to place them in an over/on a hot plate at about 150-180°C or so for about ten minutes to drive off any residual moisture that may be sticking to them, and let them oxidize just a little bit. Note that one has a '+' sign on it and one has a '-' sign. After cleaning them and heating them a bit, carefully pick them up in a paper towel (don't touch them!) and go up to the chemistry lab on the third floor and weigh them both, to the nearest thousandth of a gram. Record their weight.

Now, back in the optics lab, assemble the electrolytic cell. This is most easily done by first placing two small pieces of rubber band on one of the electrode and then sandwiching them evenly between the second electrode. While holding this together, now take two other rubber bands and bind the sandwich that you have made. Try to do so without bending/scraping the plates and without getting them too marked up with your fingerprints. When you are done binding them be sure that the plates are not touching one another, that is, not making electrical contact and that furthermore the spacing between the plates is somewhat uniform. This completes the construction of the cell electrodes.

Now place about two inches depth of tap water in the beaker. Connect wires to the cell you have, and lower them into the beaker so that their ends are in the solution.

IMPORTANT: The electrodes need not to be totally submerged. We strongly recommend that the contact points of the plates with the wires are well above the surface of the solution.

Now measure the resistance of the cell with the DVM. Record the resistance in ohms.

Finally dump out the water and refill the beaker, again to the two-inch mark or so, with the $CuSO_4$ 0.2 molar solution. Again lower the electrodes into the solution into the same general position as was done for the water resistance reading above. Measure the resistance of the cell in this solution. Record.

Now we are ready to energize the cell. WITH THE D.C. SUPPLY OFF hook up the cell and the DVM. Note that now the DVM should be set to AMPS. Now, recording the start time, you need to turn on the D.C. supply and adjust the volts until the reading on the ammeter reads about 2 amps. Watch the ammeter intently. The current will stay rather constant for anywhere between 10 seconds and a minute. Ultimately the current will start to climb precipitously upward. After 25 seconds or just when the current starts to climb, WHICHEVER COMES FIRST, record the time and shut the supply.

Now disconnect the apparatus, carefully "open" the cell by pulling out the electrodes and gingerly taking off the rubber bands. Try to avoid dropping any of the plated copper, the reddish sand that you will see clinging to the – electrode. Place the electrodes in the oven/on the hot plate again for a few minutes or so to let them dry. Carefully take them back up to the third floor chemistry lab and weigh them again. Record the new weight of both electrodes, the positive one with the Cu dust and the negative one (which will be bare).

After weighing the electrodes, etc dry, throw away the Cu dust, clean and wash the electrodes and return them to the lab and clean up a bit, making sure that the chemicals are properly stored. DO NOT RETURN THE $CuSO_4$ SOLUTION THAT YOU USED TO THE 0.2 MOLAR, but just to the "used" jar of $CuSO_4$ solution.

Conclusions:

Now that we have made all our measurements, we are ready to compute our measured value for the electron's charge. Since we recorded the current and the time that it flowed in the cell we have (by Eq. (5.5)) a measure for the total number of coloumbs of electricity that have been transported through the solution. Now, comparing the weight of the (-) electrode before and after we know how many moles of copper were deposited on the plate. Thus we can compute the number of copper atoms that represents (using Avagadro's number) and from there compute the charge of a Cu^{+2} ion (in coloumbs) by dividing the total charge by the number of copper atoms.

Q: What is then your measurement for the charge of an electron?

Q: Note the mass change of the (+) electrode. Why is that what it is?

Try to estimate the errors in your measurements of the current and the time as well as the weighings. Combine them to get a quantitative estimate of the error in your measurement of the electron's charge.

Q: What systematic effects can you think of that effect the measurement? Why was the resistance of the cell in ordinary water measured? Can you use that measurement to address one of the systematic errors in this experiment? How?

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[1] L. de Broglie, Thgse, Paris, Masson, Paris, 1924.
[2] C.J. Davisson and C.H. Kunsman, *Science* **64**, 522 (1921).
[3] W. Elsasser, *Naturwiss*. **13**, 711 (1925)
[4] N.W. Ashcraft and N.D. Mermin, Solid State Physics, Holt, Rinehart, and Wilson, 1976, Philadelphia, PA