Faraday Effect and the Determination of *e/m*

Abstract

In this experiment you will measure the rotation of the plane of polarization of light passing through glass samples in a magnetic field (the Faraday effect). By measuring the rotation, one can deduce *e/m* of the bound charges, showing them to be electrons.

I. Theory of the Faraday Effect

Optical activity is the rotation of the plane of polarization of light passing through a medium composed of "handed molecules"; it was discovered by Arago in 1811. Faraday discovered a related phenomenon in 1845: the rotation of the linear polarization of light by a refracting medium in a static magnetic field. (See Figure 1.) This was important historically as the first evidence of a connection between light waves and static magnetic fields. It eventually led to the unification of electromagnetism by Maxwell in 1865.

In both cases, the rotation of the linear polarization can be related to the difference in refractive index n for right- and left-circular polarized light. The angle of rotation ϕ is related to the path length and the difference in indices of refraction

$$\phi = (\pi\ell / \lambda)(n_L - n_R) \tag{1}$$

where λ is the wavelength and ℓ the sample thickness. The same formula can be used to describe optical activity and Faraday rotation. This rotation angle can have either sign; the sign of ϕ is tied up with the conventional definition of the magnetic field. We will not be concerned with the sign of ϕ in this experiment or the sign of the Verdet coefficient; just ignore signs!

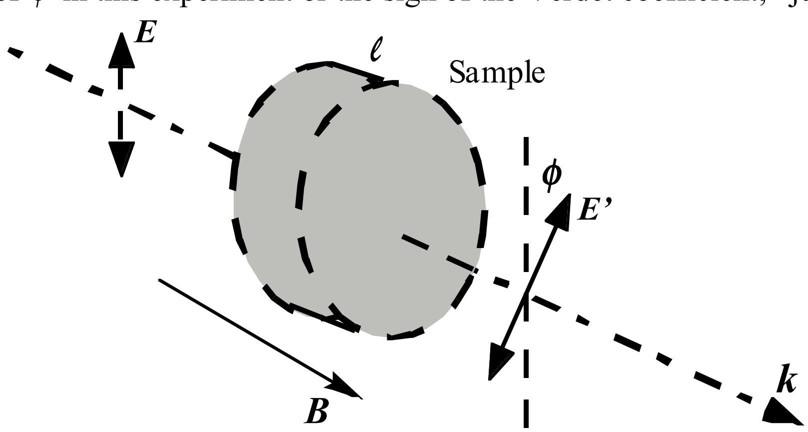

Figure 1 – Light with electric field vector $\boldsymbol{E}$ is incident from the upper left and passes through the sample that is in a longitudinal magnetic field $\boldsymbol{B}$. The polarization is rotated through an angle ϕ.

The observed Faraday effect shows a linear dependence of ϕ on the magnetic field B in the sample and on the thickness ℓ of the sample. This relationship can be described by the Verdet coefficient $\boldsymbol{V}$, defined as

$$\boldsymbol{V} = \phi / \ell B \tag{2}$$

For small magnetic fields, we can define $\boldsymbol{V}$ by the slope of ϕ *vs*. B,

$$\boldsymbol{V} \cong \left(1/\ell\right)\left(\Delta\phi / \Delta B\right) \tag{3}$$

The mechanism underlying the Faraday effect can be understood in terms of atomic structure, especially in the shift of atomic frequencies by a magnetic field. This is called the Zeeman effect, discovered in 1896, long after Faraday's work. For small fields the frequency shift is given by

$$\omega_Z = \pm eB / 2m$$

where e and m are the charge and mass of the bound electron. This result can be derived from a classical "spring and ball" model of the atom. [A proper quantum treatment happens to give the same answer.] The $\pm$ sign refers to the orientation of the magnetic field relative to the orbital angular momentum of the electron. If we assume that the frequency dependence of the refractive index is related to the frequency of the electron in the atom, then we can relate the difference in indices $\left(n_L - n_R\right)$ to the Zeeman frequency shift

$$n_R = n + \omega_z\left(dn / d\omega\right) \qquad\qquad n_L = n - \omega_Z\left(dn / d\omega\right) \tag{4}$$

where $\omega = 2\pi c/\lambda$. Therefore,

$$\left(n_L - n_R\right) = -2\omega_Z(dn / d\omega) \tag{5}$$

Expressed in terms of λ,

$$\left(n_L - n_R\right) = \left(\omega_Z / \pi c\right)\lambda^2\left(dn / d\lambda\right) \tag{6}$$

Combining equations (1), (2), (3) and (6) we find a relation for the Verdet coefficient

$$\boldsymbol{V} = \left(e / 2mc\right)\lambda\left(dn / d\lambda\right) \tag{7}$$

This relation was deduced empirically by Becquerel and derived by Larmor in 1900. It shows that $\left[2c\boldsymbol{V} / (\lambda\ dn / d\lambda)\right]$ is equal to the fundamental constant e/m and should be the same for all atoms! A measurement of the Verdet coefficient and of the wavelength dependence of the refractive index (chromatic aberration) should permit the calculation of e/m for the electron. This can be compared with the result for e/m obtained from the deflection of an electron beam by a magnetic field.

For most refracting materials, the refractive index is dispersive, decreasing as λ increases. This behavior can be fitted by the empirical Cauchy formula

$$n \cong a + b / \lambda^2 \tag{8}$$

where a and b are constants. The slope is then

$$dn / d\lambda \cong -2b / \lambda^3 \tag{9}$$

Combining Eqs. 2, 7 and 9, we can write the Verdet coefficient as

$$\boldsymbol{V} = \phi / \ell B = -2b(e / 2mc)\lambda^{-2} \tag{10}$$

which is negative and decreasing as λ increases.

The dispersion $dn/d\lambda$ can be related to the so-called Abbe constant V_D, defined as

$$V_D = \frac{(n_D - 1)}{(n_F - n_C)} \tag{11}$$

Here $n_{D,}$ $n_{F,}$ and n_C are the indices of refraction at three convenient wavelengths:

$$\lambda_D = 589.0 \text{ nm} \quad \text{(Na-D line)};$$
$$\lambda_F = 468.13 \text{ nm} \quad \text{(Fraunhofer F hydrogen line)}$$
$$\lambda_C = 656.28 \text{ nm} \quad \text{(Fraunhofer C hydrogen line)}$$

From Eq. 9,

$$-\frac{2b}{\lambda^2} \cong \lambda \frac{\Delta n}{\Delta \lambda} \tag{12}$$

From Eq. 11, we can write

$$\frac{\Delta n}{\Delta \lambda} = \frac{n_F - n_c}{\lambda_F - \lambda_C} = \frac{n_D - 1}{V_D(\lambda_F - \lambda_C)} \tag{13}$$

so

$$-\frac{2b}{\lambda^2} \cong \lambda \frac{\Delta n}{\Delta \lambda} = \frac{\lambda (n_D - 1)}{V_D(\lambda_F - \lambda_C)} \tag{14}$$

Using Eqns. 10 and 14, we can derive a value of e/m from the measured ϕ and $\boldsymbol{B}$.

II. Apparatus

- Magnet with axial hole in pole tips
- Magnet power supply (0-15 amps)
- Glass samples of known n_D and V_D with holder
- Gaussmeter
- Linear polarizer
- Linear analyzer with vernier scale
- Sodium lamp, laser
- Miscellaneous optical benches and holders, preferably non-magnetic

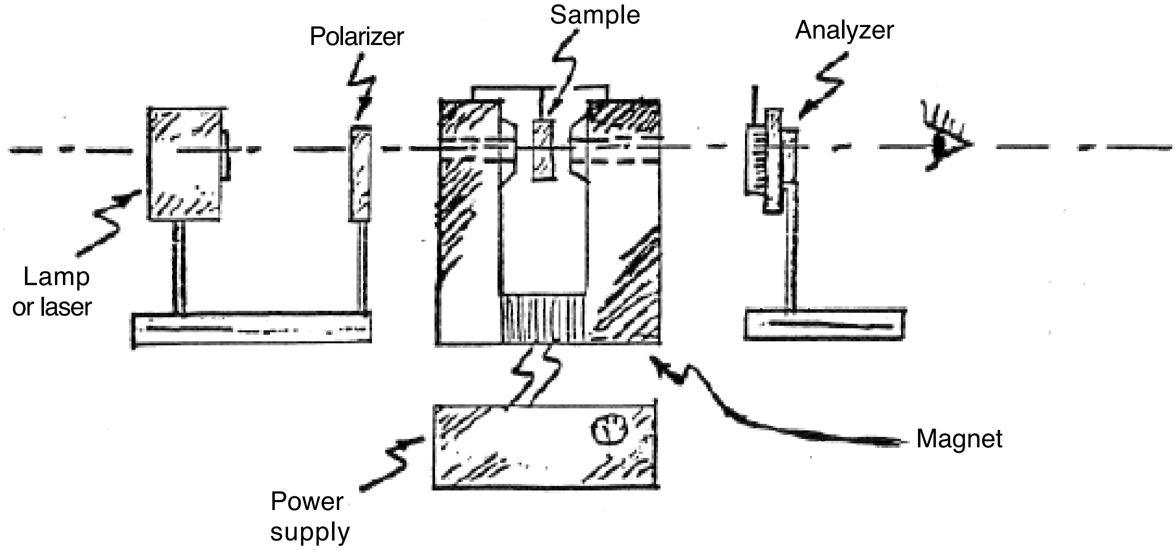

Figure 2 – General layout (Excuse the crude sketch!)

III. Procedure

The suggested setup is shown in Figure 2. DO NOT LEAVE THE MAGNET AT HIGH CURRENT SETTINGS FOR A LONG TIME, OR IT WILL OVERHEAT. DO NOT RUN THE POWER SUPPLY ABOVE 15 AMPS EVEN FOR A SHORT TIME. BEFORE LEAVING THE LABORATORY, CHECK THAT THE GAUSSMETER AND THE MAGNET POWER SUPPLY ARE TURNED OFF!

A) Calibration of the Magnet

Determine the magnetic field B *vs.* current I using a gaussmeter. Make sure that the meter is properly calibrated by checking it with the standard magnet. Estimate the uncertainties. To check for hysteresis effects in the iron, measure B for I increasing from 0 to about 14 A, and then decreasing again to 0. Plot your results B *vs.* I.

B) Determination of the Faraday Rotation

Set up the apparatus toward one end of the table with the power supply nearby so you can conveniently turn the magnet current on and off. Carefully align all the elements so that the beam goes straight down the bore of the magnet into the analyzer. If you are using a sodium lamp as a source, you may want to use a lens to focus the source onto the sample, but the lens probably isn't necessary if you put the elements close together. For a laser, of course, the lens is not necessary. Don't look directly at the laser light without a sample in the holder. Insert a sample between the pole pieces. Samples with larger V_D and greater thickness are preferred because they give larger rotation angles ϕ.

Carefully rotate the analyzer until you get extinction, and record the angle. Then turn on the magnetic field and find the new angle. Arrange the polarizer so that you can do this conveniently and quickly. You'll find that if you a careful you can do this with a repeatability ~0.5°. Take >10 measurements for each sample and average the rotation angles to get the mean and uncertainty of the mean.

C) Refining the Measurement of ϕ

The Faraday rotation ϕ for our setup is quite small, ~3°, so the experimental challenge is to measure ϕ as accurately as possible to get the best value for *e/m*. The technique described below is suggested. You are encouraged to develop other techniques to improve the measurement, especially if this is a 3-week experiment.

The intensity of the light passing through the analyzer varies as $\cos^2\theta$ where θ is the angle between the polarizer and analyzer axes; therefore, the minimum at extinction is broad, and the angle at which the minimum occurs is hard to measure accurately. To refine your measurements, use a photodiode with a digital voltmeter to read out the photocurrent. Use a shunt resistor across the voltmeter such that the voltage never exceeds 0.5 volt. Use a light shield near the photodiode to reduce stray light reaching it. Set the analyzer near 45° where the intensity varies rapidly with angle. Map the intensity *vs.* angle around 45°. Now when you turn on the magnet, you can measure the rotation by noting the new photocurrent. Use the micrometer scale to read the analyzer angles accurately.

You should do at least two samples and at least two wavelengths. The sodium lamp works well (without any filter), and you can use a low-power He-Ne laser for the other. If you have trouble with fluctuations in the He-Ne laser output, you can try a solid-state laser or LED. A laser pointer works well. Other laser or LED sources may also be available.

Be sure to record V_D, n_D, and the length ℓ, which is labeled on the samples.

Analyze these data to obtain *e/m* using Eqns. (10) and (14), and estimate the uncertainty. Make a table of your data giving: Sample, ℓ, n_D, V_D, λ, $\langle\phi\rangle$, $\langle\Delta\phi\rangle$, $\mathcal{V}$, $\lambda\dfrac{dn}{d\lambda}$, e/m.

Watch your units. It's best to use mks units consistently, and, of course, the angles are in radians. Your report should include your photodiode curve and a graph of B *vs.* I.

IV. Questions

(1) Make a reasonable average of your values for *e/m* and estimate its uncertainty. Compare this with the accepted value.

(2) Discuss the agreement (or lack of agreement) of your value with the accepted value.

(3) Estimate systematic uncertainties connected to each of the quantities that go into your calculation of *e/m*. Which error(s) dominate(s) the uncertainty in *e/m*?

(4) Would you be able to measure *e/m* with a sample for which the index of refraction did not vary with wavelength? Explain briefly the connection between the Faraday effect and chromatic aberration.

(5) If, after passing through the magnetic field, the light is reflected off a mirror and goes back through the magnet again, would the Faraday rotation be doubled or would it be 0?

V. References

Jenkins and White, *Fundamentals of Optics*, 4th edition, pages 479, 686.

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