Phys 3704L : Fine Structure of Spectra

Introduction to Fourier Transform Spectroscopy

Purpose: Measure the first fine structure interval in Sodium and Rubidium atomic spectra.

Introduction: In the non-relativisitic hydrogen atom Schrödinger equation, all the angular momentum states of a given n-level (principal quantum number) have the same energy. This is a consequence of a so-called *dynamical symmetry* of the two-body coloumb problem, called the Runge-Lenze symmetry. In many electron atoms, the Coloumb interaction between the electrons removes this degeneracy (this is called the Russel-Saunders coupling). For example, $l = 0$ (S-state electrons) are essentially falling radially inward and outward. They thus 'cut through' the cloud formed by all the other electrons and see more of the nuclues. As a result of that, they are more strongly bound than, for example, the $l = 1$ (P-state electrons) which cruise continually through the 'cloud' of the other electrons, don't get as near to the nucleus and spend more of thier time feeling the repulsion of the other electrons. This reasoning indicates that the S-state will have lower energy than the P-state even at the same n (principal quantum number), thus we expect there to be dipole (for example, optical) transitions between these S and P states. Still, one expects on rather general grounds that all the P-states of a given n level should be degenerate (i.e. have the same energy), as they differ from each other only by their spatial orientation, and space is isotropic.

But there is more to the story. As we know from the first two weeks of this class, relativity will play a role, even at these small velocities (the electrons in the hydrogen atom ground state are travelling at about 1/137'th the speed of light). Indeed, the effects of relativity (combined with another quantum attribute called 'spin,' itself also a consequence of relativity...but that is another story!) cause there to be splittings between some of these degenerate P levels. How can this be and how big are these splittings?

In this laboratory we identify a few of these nearly degenerate levels in the Sodium atom and the Rubidium atom, two atoms with 'hydrogenic' energy level schema. These atoms (and all the so-called 'alkali earths', the denizens of the the first column of the periodic chart) have one lone electron in their outermost shell, just like hydrogen. We employ a few different spectroscopic techniques to measure the energy splitting between these levels, the so called 'fine structure splitting'.

Theory: The many body effects in multielectron atoms result in energy splittings between the S- and P-hydrogenic state of an alkali earth atom. We have solved the hydrogen atom assuming that eletrons were little lumps of charge and had no other attributes (they were assumed to be 'scalars' in the parlance) besides their mass. Thus the three P-states we should have expected would be exactly degenerate since they differ only by their spatial orientation. As we will see in this lab, there are marked energy differences between these different P-states. Pauli's bold hypothesis that electrons possess spin (and have a magnetic moment alligned with the spin) was able to explain this splitting between P-states.

The physical idea is very simple; electrons are also like little bar magnets. As the electron sweeps through the electric field around a nucleus, relativity indicates that the in the electron's frame there will arise a magnetic field. Thus, the electron can line up with the field (costing energy) or anti-alligning with the field (lowering the electron's energy). Recall from what we learned about how fields transform under special relativity; the magnetic field that arises moving through the radial electric field is perpendicular to the orbital motion and electric field.

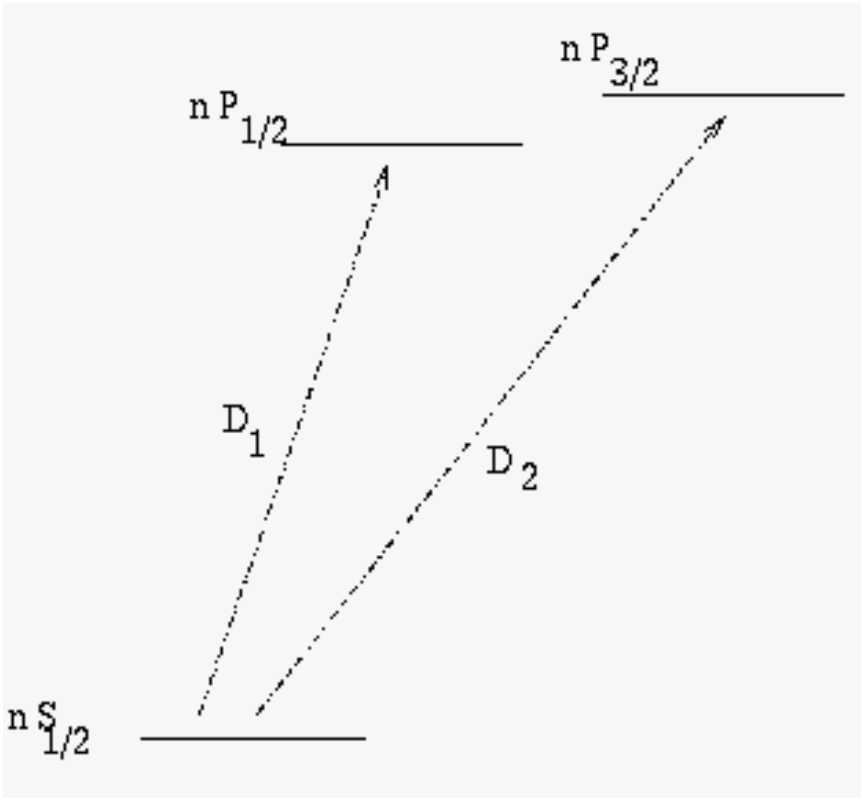
For an electron with no angular momentum (S-wave electrons), the only motion is radial and so there is no electric field perpendicular to the motion, and thus the electron sees no magnetic field.

Thus, S-wave electrons do not see any magnetic field and so do not undergo an energy shift due to this effect.

For electrons with angular momentum, on the other hand, since part of their motion is perpendicular to the radial electric field, they will experience (in their frame) a magnetic field. The electron's orientation with respect to this field will result in a difference of energy, the energy of the dipole $\vec{\mu}$ magnetic moment in a magnetic field $\vec{B}$ being $U_{dipole} = -\vec{\mu} \cdot \vec{B}$.

Finally, since magnets like to anti-allign with the field, we expect that configuration to have the least energy. By relativity, the field that arises is $\vec{B} = -\vec{v} \times \vec{E}$, which we see points in the same direction as the angular momentum of the orbit. The magnetic moment of an electron, on the other hand, points conventionally in the direction of the spin, or assumed intrinsic angular momentum, $\vec{S}$ of the electron. The total angular momentum of the orbit is thus orbital plus intrinsic, namely, $\vec{J} = \vec{L} + \vec{S}$. Thus, by this reasoning, electrons whose spin points into the $\vec{L}$ will have higher energy than electrons whose spin points opposite $\vec{L}$ (and thus the former will be less well bound, and thus emit/absorb a higher energy photon when they are decaying to/absorbing from the ground state).

Since all these alkali atoms have one lone electron outside of a closed shell of electrons, they all have S-wave ground states. Pauli surmised that electrons must have half $\hbar$ in angular momentum. Thus, the lowest lying piece of the energy level, where the levels are labeled as L_J, where as usual 'S' means $L = 0$ and 'P' means $L = 1$ (both refering to orbital angular momentu), diagram looks like,



where n is the principle quantum number of the single electron ('hydrogenic') state, and conventionally, $n = 3$ for the outermost electron for Sodium and $n = 5$ for the outermost electron of Rubidium. Here we have included the fine structure splitting due to the interaction of the electron's magnetic moment with the magnetic field it sees due to its motion. This is embodied in the diagram through the $J = L + S = 1/2$ electron and orbital angular momentum are anti-alligned (lower energy) whereas for total angular momentum $J = 3/2$ they are alligned thus (higher energy).

Since we now have atleast a crude physical picture of the cause of the splitting, we now jot down a simple suggested way to model it. The velocity of the electron is $v \sim \alpha c Z/n$ in the hydrogen

atom. The local electric field is $\vec{E} \sim Z^2 e^2/r$, but since $r \sim \frac{a_0}{Z} n^2$ and using the fact that the B field that arises is proportional to $vE \sim \#Z^4/n^3$. Thus we expect the fine structure splitting to go roughly as $\delta E \sim Z^4/n^3$.

Very naively putting in the Z (37 for Rb and 11 for Na) and the n (5 for Rb and 3 for Na) we thus expect that the *ratio* of the fine structure splitting between Rb and Na will be

$$\frac{\delta E_{Rb}}{\delta E_{Na}} = (37/11)^4/(5/3)^3 \sim 27.6 \tag{1}$$

There are many ways to critique this result (what Z effectively would the outer electrons really see, and why are we using $n = 3$ and 5 for these states...why not $n = 1$ for both if $Z_{eff} \sim 1$ from the electrostatic shielding of the nucleus by the other electrons, etc and so on and so forth ...?) but we blindly quiet ourselves, measure the ratio of the splittings experimentally and in the end compare with Eq.(1).

Proceedure: First, turn on both the Sodium lamp and the Rubidium lamp, as they take about 20 minutes to warm up and stabilize.

We will (attemp) to measure the fine structure splitting in both of these species with four different techniques.

Proceedure 1: Handheld spectrometer. Write down qualitative features of what you see.

Q: For light from the sodium lamp, can you see multiple yellow lines?

Q: For light from the rubidium lamp, can you identify which lines are occluded by interposing the hot Rb cell? One line or two?

Proceedure 2: Commercial Compact spectrometer. Use the department's ocean optics spectrometer to observe the spectra from the sodium lamp and the rubidium lamp.

Q: Record the spectra, and the lines of interest, graphically in your lab writeup.

Q: Interpose the hot Rb cell between the spectrometer and the Rb lamp. Record that spectra and use it to identify

Q: Can you resolve the Sodium D lines, or are they too close together in wavelength?

(1) which emission lines are associated with the Rb doublet.

(2) fit the data to measure the fine structure splitting in Rb. Compare with the accepted values below.

Proceedure 3: Fabry-Perot cavity. IMPORTANT: this one takes a bit of time...make sure to go to proceedure 4 first and get it running...you can then leave it while it is running and do this part 3

Follow the proceedure as described in the following 3 pages from the lab description of Irina Novikova of William and Mary University.

NOTE: If you are not using the diffusing plate, set up your cavity about 12 feet or so from the sodium lamp, and move your head or apparatus to have the bullseye pattern form at a location other than at the bulb.

Q: Independently, using calipers and not light, identify how far the mirror moves per division on the micrometer.

Q: Count a fixed number of fringes going by and determine the average wavelength, $\bar{\lambda}$, of the sodium D-lines (yellow light) using the calibration you completed in the preceeding question.

Proceedure 4: Fourier Transform Spectroscopy: Familiarize onself with the operation of the Michelson interferometer. (see attached pages) Our's is instrumented with a synchronous motor that slowly steps through the fringe pattern. The output of the interferometer is fed to a photodiode whose output is recorded on the computer.

You need to read the following pages, setup the interferometer (it is a bit tricky to align!) and get the system collecting data continuously while you do the other part (Proceedure 3) of the lab.

Analysis and reporting:

1) Answer all the questions under proceedure 1 and 2, for both lamps.

2) Determine the average wavelength of the sodium D light using the calibration for proceedure 3 and find the difference of the caliper readings as you wind through a given number of fringes.

3) Follow through the rest of the analysis for the Fabry-Perot cavity as described near the end of proceedure 3.

4) Take the fast fourier transform of the data retrieved from proceedure 4. Using the known value for the wavelength of the first line only and your fourier transform, determine the wavelength of the other line(s) and thus compute the fine structure splitting in sodium.

5) Use your data to compute the ratio of the fine splittings (in energy) and compare what you find experimentally with the naive theory of Eq.(1).

For the curious, here is a summary table of spectroscopic data of the alkalis; See if they also fit into the ratios in Eq.(1).

$atom$	n	Z	$D_1(in\ nm)$	$D_2(in\ nm)$
Li	2	3	670.992	670.977
Na	3	11	589.5924	588.9950
K	4	19	769.9	766.7
Rb	5	37	794.7671	780.2412
Cs	6	55	894.5930	852.3473