Temperature Dependence of Sound Speed

Physics 3705: Spring 2025

Objective: Quantify the magnitude and exponent of the temperature dependence of the speed of sound in air, as a test of the kinetic theory of gas.

Theory: The speed of sound in a medium depends on the underlying physics of the elastic response (and the mass density) of the medium in which it travels.

$$v = \sqrt{B/\rho} \qquad \text{where} \qquad 1/B = -\frac{1}{V}\frac{\partial V}{\partial P}|_X \tag{1}$$

where B is the bulk compressibility with 'X' fixed. Most of the times in a solid, the compressions and rarefaction making up the sound wave do not really lead to prompt temperature changes, so that we may take for solids X to be temperature. For gasses at that is typically not the case. Instead, in gasses, the local compressions/rarefactions for typical acoustic frequencies are associated with prompt temperature changes. Thus, the parcels of air at different places in the wave are not thermally connected to one another. Since there is not enough time for heat to enter or leave each parcel of air, the process is nearly **adiabatic** (so $X = s$, the local entropy density).

During an adiabatic process, we have PV^γ being constant, so that $P\rho^{-\gamma}$ is a constant, so $1/B = \frac{1}{\rho}\frac{\partial\rho}{\partial P}|_S = \gamma/P$. For a monoatomic gas $\gamma = 5/3$ whereas for a diatomic gas we find $\gamma = 7/5$. In Eq.(1) this leads to

$$v = \sqrt{\frac{\gamma P}{\rho}} \tag{2}$$

Approximating the air's equation of state with that of an ideal gas, we have $PV = Nk_BT$ which we can write as $P = \frac{k_B}{\bar{m}}\rho$ where $\bar{m}$ is the mean mass of the particles making up the air. Putting this last relation into Eq.(2) then yields,

$$v = \sqrt{\frac{\gamma k_B}{\bar{m}}}\sqrt{T} \tag{3}$$

At 26^o, the speed of sound in air is 343 m/s. We can use that to numerically estimate the mean mass $\bar{m}$ from Eq.(3) to be $\bar{m} = 4.91 \text{ x } 10^{-26}$Kg. For comparison note that for nitrogen, N_2, one has 28 AMU = $4.65 \text{ x } 10^{-26}$Kg.

We don't measure the speed v directly; instead we measure the frequency of the dominant note of a flute of fixed length, l, which is strictly proportional to v,

$$f = \alpha v/l \tag{4}$$

where α is a constant near 1/2. We can then test the temperature dependence expected via kinetic theory through Eq.(3) using the measured frequencies and temperature two ways. First test if fractionally, at room temperature (RT)

$$\frac{\delta v}{v}|_{RT} = \frac{\delta f}{f} = \beta\frac{\delta T}{T}|_{RT} \tag{5}$$

That is, compare the slope β at room temperature to that expected by kinetic theory. To do this fit all your temperature data with a straight line.

Next, eschew the linear fit above and fit a curve of $v(T)$ to the function $a + bT^c$ where a, b, c are constants. From the fit you will be able to determine whether your data is consistent with Eq.(3) in that you can ask if your determination of c is within the expected value.

Method:

1) Download and configure the app "phyphox" on your cell phone. Make sure that you can operate the 'audio spectrum' tab. Go to the 'Settings' tab of the audio spectrum section of the app and set "samples" to 32768 (sets the frequency resolution).

2) Practice blowing the same through the flute each time. You can use the "Audio Autocorrelation" tab to practice blowing into the flute in a uniform way so that each time you get nearly the same frequency.

3) Attach the microthermistor (that you already calibrated) to the inside of the flute and connect it to the meter for continuous recording of the air temperature inside the flute.

4) At room temperature take (and save) three recordings of the flute frequency spectrum. Then cool the flute down in the icebox and retrieve it and make a few recordings of the frequency spectrum as a function of the thermistor reading (air temperature in the flute). Finally heat the flute up with the devil's hairdryer and again record frequency spectra as the temperature comes back to room temperature.

Analysis:

5) Fit each frequency spectrum dataset with a gaussian to extract the center frequency for each dataset. Then plot those frequencies versus temperature and fit with a straight line. Does it agree with the expected value?

6) Finally fit your center frequencies data with $a + bT^c$, where T is the absolute temperature to extract c and the error in your determination of c. Does it agree with the expected value?