MEASUREMENT OF e/k_B

Background and Theory:

In his article, "A Simple Laboratory Experiment to Measure e/k", Fred Inman detailed a relatively quick and easy method to determine the ratio of the electron charge to Boltzmann's constant, e/k_B[1]. The author devised the apparatus shown in figure 1 below where a DC voltage source was connected to a transistor by way of a potentiometer. The voltage difference across the emitter and base and the current through the collector were measured with digital multimeters. The purpose of this project was twofold. The first part was to verify that the ratio e/k_B could be accurately measured with the setup outlined in figure 1. The second was to determine the barrier height, ϕ_B, of the transistor.

Figure 1

The current through the collector of the transistor under general loading is given by:

$$I = C_1 T^2 \exp\left(\frac{-e\phi_B}{k_B T}\right)\left[\exp\left(\frac{eV}{nk_B T}\right) - 1\right]$$

where C_1 and n are constants that depend upon the transistor. In this case n is equal to one since the semiconductor junction is not externally biased. Also if V is sufficiently large, $1 \ll \exp\left(\frac{eV}{nk_B T}\right)$, the negative one can be neglected. By utilizing these simplifying assumptions, rearranging the equation, and taking the natural logarithm of both sides one can obtain:

$$\frac{I}{T^2} = C_1 \exp\left(\frac{e}{k_B T}(V - \phi_B)\right)$$

$$\ln\left(\frac{I}{T^2}\right) = C_2 + \frac{e}{k_B T}(V - \phi_B)$$

where $C_2 = \ln(C_1)$ and is presumably a constant that is dependent upon doping and the geometry of the junction but not temperature or voltage. Differentiating the above relation with respect to V while holding T constant will yield:

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[1] Inman, Fred. "A Simple Laboratory Experiment to Measure e/k". *The Physics Teacher*. Volume 43, January 2005. pgs 27-28.

$$\frac{d}{dV}\left[\ln\left(\frac{I}{T^2}\right)\right] = \frac{e}{k_B T}$$

From this it is obvious that plotting $\ln\left(\frac{I}{T^2}\right)$ versus V at a constant temperature will result in a line with a slope of $\frac{e}{k_B T}$. Therefore if the absolute temperature of the transistor is known $\frac{e}{k_B}$ can be calculated.

If instead of differentiating with respect to V the relation is differentiated with respect to $1/T$ while holding V constant the derivative will become:

$$\frac{d}{d(1/T)}\left[\ln\left(\frac{I}{T^2}\right)\right] = \frac{e}{k_B}(V - \phi_B)$$

A plot of $\ln\left(\frac{I}{T^2}\right)$ versus T at a constant applied voltage is known as a Richardson Plot[2] and results in a line with a slope of $\frac{e}{k_B}(V - \phi_B)$. Since the value of $\frac{e}{k_B}$ was already determined ϕ_B can be readily computed.

Test Procedure:

The apparatus used to perform the test is shown in figures 2 and 3. It is a slightly modified version of the circuit arrangement used by Inman. Instead of using a potentiometer, two variable resistors, R_1 and R_2, were used to vary the voltage across the transistor. Additionally, the transistor was insulated in order to collect current data as a function of temperature.

The construction of the apparatus was facilitated with the use of a breadboard. For the voltage source a typical 1.5 volt battery was used. The voltage across the transistor was controlled with two dial type variable resistors with a range of zero to 10 000 Ohms. The temperature of the transistor was measured with a thermistor wrapped around it and placed in a copper tube to ensure a uniform temperature distribution. The copper tube was then inserted into a styrofoam block. Two digital multimeters were then used to measure the voltage and the current through the circuit.

Data was first collected at a constant room temperature with a varying voltage in order to determine e/k_B. Two transistors were used. The first was an n-p-n type germanium transistor and the second was an n-p-n silicon transistor. In turn, each transistor was connected to the test circuit. The resistances of R_1 and R_2 were then varied to achieve a

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[2] Schroder, Dieter K. *Semiconductor Material and Device Characterization 2nd Ed.* John Wiley & Sons, Inc. New York, New York. 1998.

range of voltages across the transistor. Each voltage was recorded as was the corresponding collector current. The transistors started off at room temperature and since they consume very little power their temperatures were assumed to be constant; furthermore, the insulating material was removed for this test to allow for the maximum amount of heat dissipation. During the test, the temperature of the room was recorded.

The second series of tests involved fixing the voltage and varying the temperature of the transistor. This was done by cooling the transistor in a freezer. The transistor, thermistor, copper tubing, and insulating material were all cooled in the freezer together. Once they had sufficiently cooled they were quickly reassembled back into the circuit. The voltage across the transistor was recorded and since the voltage was to remain constant, the voltage measuring multimeter was disconnected from the circuit and used to measure the resistance of the thermistor. The transistor was then allowed to slowly warm up to room temperature; a process that took about two hours. Meanwhile the collector current and the thermistor resistance were measured with the multimeters and recorded once a second on a computer with Meterview software. The resistance was later scaled to temperate. This procedure was performed at two different voltages for each transistor.

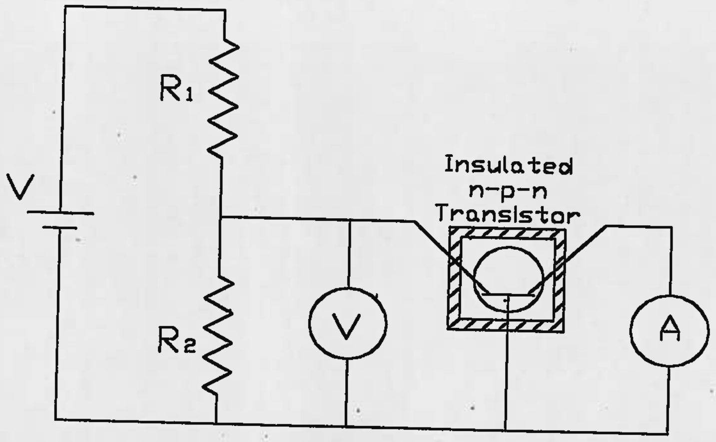

Figure 2

Figure 3

Results and Conclusions:

For the constant temperature data $\ln\left(\frac{I}{T^2}\right)$ was plotted against V. These data points were then fitted with a line. The slope of the line is a measurement of $\frac{e}{k_B T}$. So knowing the absolute temperature allows the value of e/k_B to be calculated. The results both the germanium and the silicon transistors are shown in figures 4 and 5.

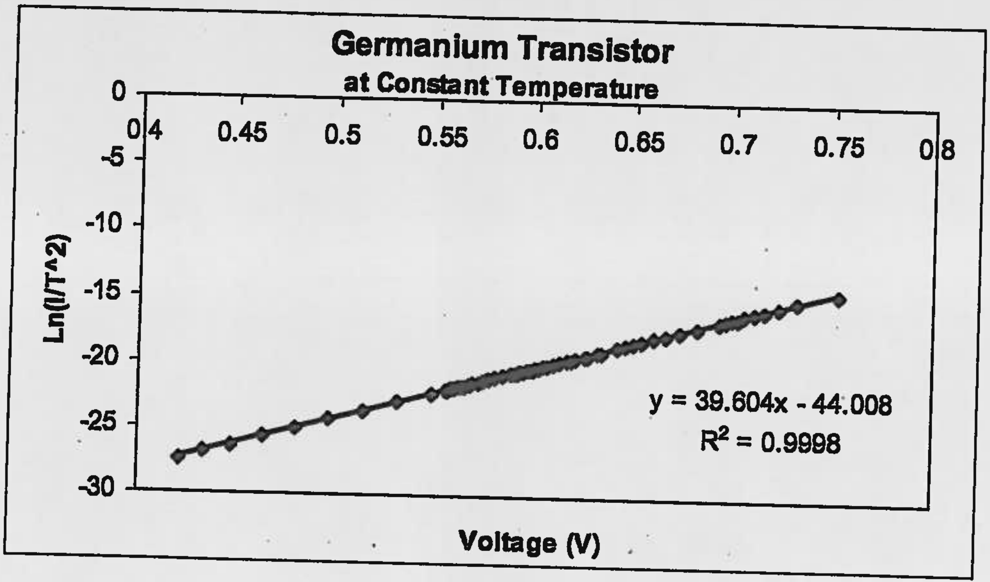

Figure 4

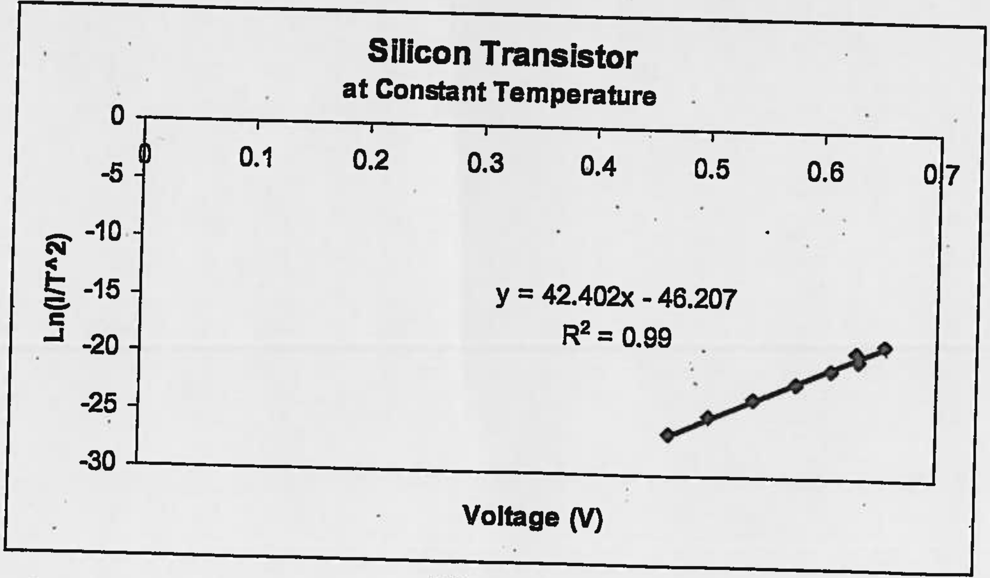

Figure 5

The values obtained for e/k$_B$ are $1.167*10^4$ K/V for germanium and $1.264*10^4$ K/V for silicon. The accepted value for e/k$_B$ is $1.160*10^4$ K/V. The standard deviations were 19.5 and 514 K/V for the germanium and silicon transistors, respectively. The value for the silicon transistor lies within two standard deviations of the accepted value. The number obtained for the germanium transistor does not; however, the deviation for this transistor is insignificant to the uncertainty in the temperature measurement. This verifies that the ratio e/k$_B$ can indeed be determined with the method described by Inman. The relatively large deviation in calculating the ratio for the silicon transistor can be attributed to the small number of data points used. Only eight points were used for the silicon transistor whereas more than thirty points were found for the germanium transistor.

The plots of $\ln\left(\frac{I}{T^2}\right)$ versus T at constant voltages are shown in figures 6 and 7. The value of $\frac{e}{k_B}$ was already determined so ϕ_B can be computed from the slopes of the plots with the equation:

$$\phi_B = V - \frac{d}{d(1/T)}\left[\ln\left(\frac{I}{T^2}\right)\right]\left(\frac{k_B}{e}\right)$$

where the differential term is just the slope of the curves in the Richardson plots below. The values of ϕ_B and their respective standard deviations obtained for the germanium and silicon transistors at both voltages are tabulated in table 1.

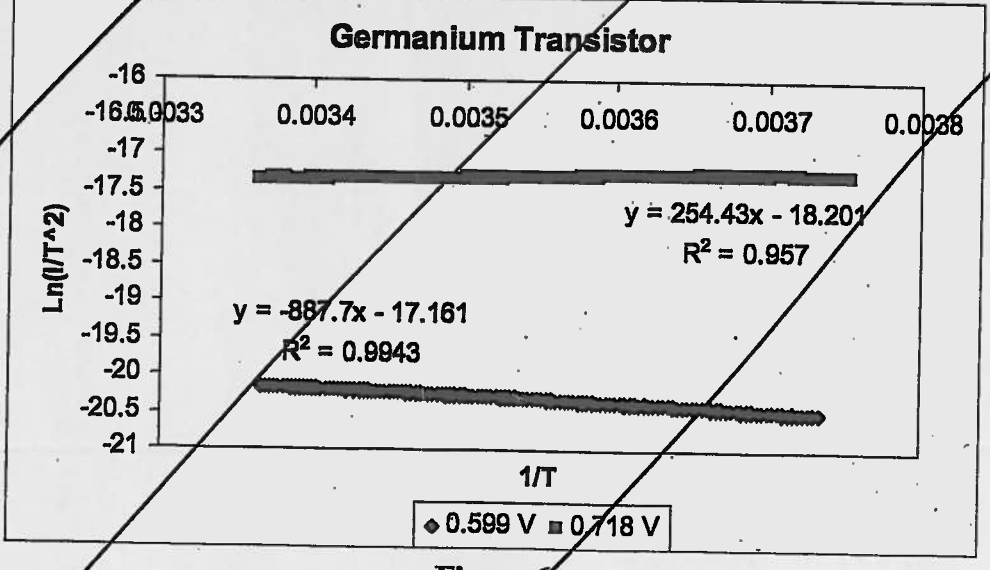

Figure 6

COMPUTE THE e/k_B FOR A COHORT OF TRANSISTORS.